MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATIONS FOR THE DEGREE IN BACHELOR OF EDUCATION (SCIENCE)

SMA 261: PROBABILITY & STATISTICS II

Date: 4/8/2016 **Time: 2:00 – 4:00 pm**

Answer question ONE and any other TWO questions

QUESTION ONE

(a) (i) Determine the $E(x_1 + x_2)$ from the Bivariate table below. (3 marks)

		x_2	
		0	1
x_1	0	$\frac{1}{6}$	0
	1	⅓	⅓
	2	0	$\frac{1}{6}$

(ii) Given y_1 and y_2 have the joint probability density function (pdf) given by

$$f(y_1, y_2) = \begin{cases} K(1 - y_2), & 0 \leq y_1 \leq y_2 \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- Determine the value of the constant K. (4 marks)
- Determine $p(y_1 \leq \frac{3}{4}, y_2 \geq \frac{1}{2})$ (5 marks)

(b) If the joint probability function for x_1 and x_2 is given by $f(x_1, x_2) = \frac{1}{30}(x_1 + x_2)$

$x_1 = 0, 1, 2, 3.$

$x_2 = 0, 1, 2$

(i) Determine marginal distribution of x_1 and x_2 (6 marks)

(ii) Determine an expression $\emptyset(x_1, x_2)$ the conditional distribution of x_1 given x_2 takes on all values of x_2. (3 marks)

(c) Consider the Bivariate table below

		y = 0	y = 1
x	0	1/8	0
	1	2/8	1/8
	2	1/8	2/8
	3	0	1/8

(i) Determine $E(x)$ (2 marks)

(ii) Determine the covariance of x and y (4 marks)

(iii) The marginal probability mass function of y (3 marks)

QUESTION TWO

(a) Let $y_1 \sim B(n_1, p)$ and $y_2 \sim B(n_2, p)$. Determine the moment generating function of $y_1 + y_2$ given that y_1 and y_2 are independent binomial random variables. (7 marks)

(b) If x and y have joint probability density function $f(x, y) = \frac{8x^2}{7y^3}$ for $1 \leq x, y \leq 2$

(i) Determine the marginal distribution of x. (4 marks)

(ii) Determine the conditional density function of x given $y = y$ (6 marks)

(iii) Determine whether x and y are independent. (3 marks)

QUESTION THREE

(a) Given that $f(x, y) = \begin{cases} 2x, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine $p(0 < x < ⅓, ½ < y < 1)$ (7 marks)

(a) The joint pdf of x and y is given by $f(x, y) = \begin{cases} 2e^{-x-y}, & 0 \leq x \leq y \leq \infty \\ 0 & \text{elsewhere} \end{cases}$

(i) Determine $f_1(x)$ and $f_2(y)$ (10 marks)

(ii) Show that x and y are independent. (3 marks)

QUESTION FOUR

The joint pdf of x and y is given by that $f(x,y) = \; 6x^2y, \begin{cases} 0 < x < 1, 0 < y < 1 \\ 0 \qquad\qquad \text{elsewhere} \end{cases}$

Determine: (i) $f_1(x)$ and $f_2(y)$ (6 marks)

(ii) μ_x and μ_y (4 marks)

(iii) $d_x^1 d_y^2$ (4 marks)

(iv) The correlation coefficient between x and y (6 marks)

QUESTION FIVE

(a) Let y be a continuous random variable with a pdf defined by that

$$f(y) = \begin{cases} 2y, & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the density of $x = 8y^3$. (8 marks)

(b) Given that $E(Y/X) = \alpha + \beta x$ where E denotes the expectation, X and Y are random variables and α and β are constants;

Show that $\beta = \dfrac{E(x,y) - E(x)E(y)}{E(x^2) - [E(x)]^2} = \dfrac{Cov(x,y)}{Var(x)}$ (12 marks)