MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 404: COMPLEX ANALYSIS II

DATE: 24/7/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

ANSWER ALL QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) State without proof the Residue theorem (2 marks)

b) Determine all the poles of the function $f(z)=\frac{z+2}{z^2\left(z^2+9\right)}$ stating the order of each pole (4 marks)

c) Evaluate the integral $\int_c \frac{5z-3}{z(z-3)}dz$, $|z|=3$ using the Residue Theorem (5 marks)

d) Find the transformation that maps the cycloid whose parametric equations are $x = a(t-sint)$, $y=a(1-\cos t)$ into a straight line (5 marks)

e) Show that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+16)^2}dx = \frac{\pi}{8}$ (5 marks)

f) Consider the function $u\,(x,y) = x^3-3xy^2$

i) Show that $u\,(x,y)$ is harmonic (2 marks)

ii) Determine the conjugate harmonic of $u\,(x,y)$ (5 marks)

iii) Write down the corresponding analytic function $f(z) = u(x, y) + iv(x, y)$ (2 marks)

QUESTION TWO (20 MARKS)

a) i. State the Schwarz-christoffel transformation (4 marks)

ii. Show that when $x_n = \infty$,the last factor in the Schwarz-christoffel transformation is 1.

(7 marks)

b) State without proof the Schwarz reflection principle. Hence determine whether $f(z) = z^2 + 1$ Satisfy the Schwarz reflection principle or not. (9 marks)

QUESTION THREE (20 MARKS)

a) Evaluate the following integrals

i. $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta$ (5 marks)

ii. $\int_0^{\infty} \frac{1}{(x^2 + 1)\sqrt{x}} dx$ (5 marks)

b) Define analytic continuation (3 marks)

c) Prove that $f_2(z) = \frac{1}{1+i}\sum_{n=0}^{\infty}\left(\frac{z+i}{1+i}\right)^n$ is an analytic continuation of $f_1(z) = \sum_{n=0}^{\infty} z^n$,

showing graphically the regions of convergence of the series. (7 marks)

QUESTION FOUR (20 MARKS)

a) Differentiate between conformal mapping and isogonal mapping (3 marks)

b) Show that the mapping $f(z) = z^2$ is conformal at the point $z = 1 + i$ with $C_1: x = 1$ and $C_2: y = i$. Find the angle of rotation and sketch the curves. (10 marks)

c) Consider triangle A, B, C bounded by the curves $c_1 : x = 1$, $c_2 : y = 1$ and $c_3 : x + y = 1$

Determine the region of the w-plane into which the above triangle is mapped by $f(z) = z^2$ .Is $f(z)$ conformal at (1, 1)? (7 marks)

QUESTION FIVE (20 MARKS)

a) Show that the function $u(x, y) = x^2 - y^2 + 2y$ is harmonic (4 marks)

b) Prove that the transformation of $u(x, y) = x^2 - y^2 + 2y$ is also harmonic under the transformation $z = w^3$ (10 marks)

c) Write the parametric equation of the ellipse $\frac{x^2}{a} + \frac{y^2}{b}$ hence find the transformation that maps the ellipse $\frac{x^2}{a} + \frac{y^2}{b}$ in the z plane onto the real axis of the w plane (6 marks)