MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCES)

SAC 406 COMPUTATIONAL FINANCE

DATE:25/7/2023 **TIME:8:00-10:00 A.M**

INSTRUCTIONS TO CANDIDATES.

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain why buying a put is not the same as selling a call. (3 marks)

b) Explain why a European put option is exercised only if $TS < K$ (2 marks)

c) Suppose that the price of Share X is 112 and that a put option on Share X with an exercise price of 110 is currently priced at 5. Calculate the intrinsic value and time value of the option. (2 marks)

d) A three-month forward contract exists on a zero-coupon corporate bond with a current price per £100 nominal of £42.60. The yield available on three-month government securities is 6% pa effective. Calculate the forward price. (2 marks)

e) Explain what is meant by a 'complete market'. (2 marks)

f) Give two reasons why in practice financial markets may not be complete (2 marks)

g) State the relevance of the concept of complete markets in derivative pricing. (2 marks)

QUESTION TWO (20 MARKS)

a) For each of the Greeks Δ , ν , Θ , ρ and λ , discuss whether its value will be positive or negative in the case of:

   a. a call option

   b. a put option. (4 marks)

b) A call option has a price of 20.15p and a delta of 0.558 at time t . Determine the hedging portfolio of shares and cash for this option at time t , given that the price of the underlying share, S_t = 240p . (4 marks)

c) Define what is meant by martingale give three features on which its relies upon (4 marks)
List the desirable characteristics of interest rate models. (4 marks)

d) Discuss the limitations of one-factor models of interest rates. (4 marks)

QUESTION THREE (20 MARKS)

a) Consider an asset with price S_t at time t , paying a dividend at a constant dividend yield, D . Dividends are paid at the end of each year and are immediately reinvested in the asset. The continuously compounded risk-free rate of interest is r pa. Derive the forward price, K, of a contract issued at time t, with maturity at time T, to trade one unit of the asset, where T- t is an integer number of years. State any assumptions you make. (6 marks)

b) By constructing two portfolios with identical payoffs at the exercise date of the options, derive an expression for the put-call parity of European options on a share that has a dividend of known amount d payable prior to the exercise date. (8 marks)

c) Let p_t be the price of a European put option exercisable at time T with a strike price K on an underlying non-dividend-paying share with price S_t at time t .

a. By considering a suitably chosen notional portfolio or portfolios (which should be specified carefully), show that the price $p_t$satisfies the inequality:

$$p_t \geq Ke^{-r(T-t)} - S_t$$ (4 marks)

b. Explain how you would modify your inequality if you knew that holders of the share on the day before the option expires are entitled to receive a cash dividend of 0.02 TS payable at time T. (2 marks)

QUESTION FOUR (20 MARKS)

a) An investor claims to be able to value an unusual derivative on a non-dividend-paying share using the pricing formula:

$$V_t = S_t^2 e^{-4s_t}$$

where S_t denotes the price of the share at time t .

i. Derive formulae for the delta and gamma of the derivative, based on the pricing formula above. (4 marks)

ii. For each of the following scenarios, calculate the number of shares that must be purchased or sold along with a short holding in one derivative, in order to achieve a delta-hedged portfolio:

A. the current share price is 1 (2 marks)

B. the current share price is 3. (2 marks)

iii. Explain which of the scenarios in (b) is likely to involve more portfolio management in the near future if the investor is determined to maintain a delta-hedged portfolio. (2 marks)

b) Suppose that the current time corresponds to t = 5 and that the force of interest has been a constant 4% pa over the last 5 years. Suppose also that the force of interest implied by current market prices is a constant 4% pa for the next 2 years and a constant 6% pa thereafter. If T = 10 and S = 15 , write down or calculate each of the four quantities P(t, T), r (t) , f(t, T,S) and r(t,T) (4 marks)

a) Explain what is meant by a short rate model of interest rates and how such models can be used to price interest rate derivatives. (3 marks)

b) Explain what is meant by a one-factor short rate model of interest rates. (3 marks)

QUESTION FIVE (20 MARKS)

a) To fund an expansion in its operations, a company has just issued 5-year zero-coupon bonds with a total face value of £10 million, taking its total asset value up to £15 million.

i. Explain how the value of the bonds can be expressed in terms of a European put option. (4 marks)

ii. Hence calculate the fair price of a holding of the company bonds with a face value of £100 using the Black-Scholes model, given that the price of a 5-year zero-coupon government bond is £77.88. Assume that the annualized volatility of the company's assets over the 5-year period is 25%. (6 marks)

iii. Explain what is meant by a credit spread and calculate its value for the company bonds. (4 marks)

b) What is meant by saying that the process $\{Y_t\}$ is a martingale with respect to another process X_t? (3 marks)

Let Bt ($t \geq 0$) be a standard Brownian motion.

c) Show that Bt and $B_t^2 + kt$ are both martingales with respect to Bt , for a suitably chosen value of the constant k , which you should specify. (3 marks)