MACHAKOS UNIVERSITY

University Examinations 2022/2023

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SMA 260: PROBABILITY AND STATISTICS I

DATE: 7/3/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Attempt Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Define a random variable. (1 mark)

b) Consider the random variable x with the cumulative distribution function (c.d.f),

$$F(x) = \begin{cases} 1 - e^{-x} & ; \ x \geq 0 \\ 0 & ; \ x < 0 \end{cases}$$

i. What is the p.d.f of x? (2 marks)

ii. Use the p.d.f obtained to find the $p(x > 10)$. (2 marks)

iii. Use the c.d.f to find the $p(x >- 8)$. (2 marks)

c) Let x have the p.d.f, $f(x) = \begin{cases} 2(1-x) \,; 0 < x < 1 \\ 0 \quad ; \ elsewhere \end{cases}$. Obtain $E(6x + 3x^2)$.

(3 marks)

d) As part of an air pollution survey, an inspector decides to examine the exhaust of 12 out of 36 vehicles of a company. If 8 vehicles of the company emit excessive amount of pollutant, determine the probability that 3 of such vehicles will be included in the sample examined by the inspector. (3 marks)

e) A fair six faced die is tossed once. Determine:

i. Its probability distribution. (2 marks)

ii. The mean and variance. (2 marks)

f) The operational lifespan of a given brand of desktop computers has been found to be normally distributed with a mean of 4.8 years and a standard deviation of 1.6 years.

i. Determine the proportion of the desktop computers that will have a lifespan of between 3.8 years and 6.6 years. (2 marks)

ii. If these desktop computers have a warranty period of 2 years, determine the proportion of original sales which will require replacement through this warranty. (2 marks)

iii. If the manufacturer of these desktop computers wants only 5% of the computers to be replaced through this warranty, determine the warranty period that should be set to achieve this. (2 marks)

g) Given the mgf of a random variable x as $e^{2(e^t-1)}$;

i. Prove that the x has mean equal to the variance. (4 marks)

ii. Write down the p.m.f of x and prove that it satisfies the conditions necessary for a p.m.f. (3 marks)

QUESTION TWO (20 MARKS)

a) A continuous random variable x is given by the function, $f(x) = \begin{cases} \frac{2}{9}(9x - x^2 - 18) \ ; \ 3 \leq x \leq 6 \\ \quad 0 \quad ; \quad elsewhere \end{cases}$

I. Show that the function is a probability density function. (2 marks)

II. Determine;

i. The mean of x. (2 marks)

ii. The variance of x. (3 marks)

iii. The median of x. (3 marks)

III. Determine the probabilities:

i. $P(x \geq 4)$ (2 marks)

ii. $P(4 \leq x \leq 5)$ (2 marks)

iii. $P(x \leq 5)$ (2 marks)

b) Suppose x has a pmf, $f(x) = \begin{cases} cx \ ; \ x = 1,2,\ldots,10 \\ \ 0 \ ; \ elsewhere \end{cases}$.

i. What is the value of c? (2 marks)

ii. $P(x \leq 2)$ (2 marks)

QUESTION THREE (20 MARKS)

a) A discrete random variable x has a probability function given by

X	0	1	2	3	4	5
f(x)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{2}{20}$

i. Show that the function f(x) is a probability mass function. (2 marks)

ii. Determine the mean and the variance of x. (3 marks)

b) A researcher claims that only 50% of the high school students capable of passing a national examination eventually pass. Assuming that this claim is true, determine the probability that among a random sample of 8 high school students capable of passing the national examination:

i. Exactly 4 will pass. (2 marks)

ii. At least 6 will pass. (4 marks)

iii. At most 3 will pass. (3 marks)

iv. Between 3 and 6 inclusive will pass. (2 marks)

c) Suppose that 10 percent of the probability for a certain distribution that is $N(\mu, \sigma^2)$ is below 60 and that 5 percent is above 90. What are the values of μ and σ? (4 marks)

QUESTION FOUR (20 MARKS)

a) Consider the random variable x with p.d.f $\begin{cases} \left(\frac{1}{2}\right)^{x-1}\left(\frac{1}{2}\right); & x = 1,2,3,\ldots \\ 0 & ; otherwise \end{cases}$

i. What is the $p(x > 5)$? (3 marks)

ii. Find the mgf of x. (3 marks)

iii. Use the mgf to compute the mean and standard deviation of x. (5 marks)

b) Given a random variable x with p.m.f, $f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & ; x = 0,1,2,\ldots \\ 0 ; otherwise \end{cases}$ Suppose we define a new random variable y such that $Y = 4x$. Determine the p.m.f of y. (3 marks)

c) Suppose x is a continuous random variable having an exponential probability distribution with p.d.f given by, $f(x) = \begin{cases} \lambda e^{-\lambda x} & ; \; 0 \le x \le \infty, \; \lambda > 0 \\ 0 & ; \quad otherwise \end{cases}$.

i. Derive the moment generating function of f(x). (3 marks)

d) Derive the first and second moment about the origin. (3 marks)

QUESTION FIVE (20 MARKS)

a) It has been observed that a certain road has 1 pot hole for every 60 metres along the road. A random length of 360 metres is selected from a section of the road.

i. Derive the Poisson probability mass function for this distribution. (2 marks)

ii. Determine the probability that from the length selected, there are between 5 and 8 pot holes inclusive. (5 marks)

iii. Determine the probability that from the length selected, there are at least 4 pot holes. (4 marks)

b) Following public outcry of teenage pregnancy in secondary schools, pregnancy test was to be carried out to students in a certain girl's school with a total population of 80 girls. A total of 12 girls were pregnant. A random sample of 20 girls was taken for testing in which the result was categorized as positive (pregnant) and negative (not pregnant). Assuming a hyper-geometric probability distribution, determine each of the following.

i. The probability that 4 of the girls sampled tested positive. (2 marks)

ii. The mean and the variance of the random variable x. (3 marks)

c) List any four characteristics of a normal distribution curve. (4 marks)