MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FIRST YEAR FIRST SEMESTER SUPPLEMENTARY EXAMINATION FOR THE BACHELOR OF EDUCATION (SCIENCE)/BACHELOR OF SCIENCE IN MATHEMATICS AND BACHELOR OF SCIENCE TELECOMMUNICATION

SPH 103: MATHEMATICS FOR PHYSICS

DATE: 26/7/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

Question ONE (Compulsory)

a) The following set of data refers to the amount of money in £s taken by a newsvendor for 6 days. Determine the mean, median, and modal values of the set:

$\{27.90, 34.70, 54.40, 18.92, 47.60, 39.68\}$ (3 marks)

b) Find $f^{-1}(x)$ when $f(x) = 2x + 1, x \in \Re$ (3 marks)

c) Given the two displacements; $\vec{D} = (6.0\hat{i} + 3.0\hat{j} - 1.0\hat{k})$ m; and $\vec{E} = (4.0\hat{i} - 5.0\hat{j} + 8.0\hat{k})$ m,

(i) Find the magnitude of the displacement $2\vec{D} - \vec{E}$. (2 marks)

(ii) Find the angle between the vectors. (2 marks)

d) Given that $y = x^5 + 2x$, find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$. (2 marks)

e) Use the chain rule to find $\frac{dy}{dx}$ when $y = (x^2 - 2)^4$. (3 marks)

f) Solve the equation $x^2 + 8x + 10$ by completing the square method. (4 marks)

g) Find the points on the curve with equation $y = x^3 + 6x^2 + 5$ where the value of the gradient is -9. (3 marks)

h) Evaluate $\int_0^2 \frac{dx}{4 + x^2}$. (3 marks)

i) Show that $\cosh 3x = 4\cosh^3 x - 3\cosh x$. (3 marks)

j) Find the magnitude of the vector $\vec{A} = 2i - 5j + 3k$. (2 marks)

Question Two

a) Write down the general form of a complex number. Explain the parts. (2 marks)

b) With the help of an Argand diagram, distinguish between the modulus and argument of a complex number. (4 marks)

c) Find the modulus and argument of the complex number $-1 + \sqrt{3i}$. Sketch the argand diagram. (4 marks)

d) Given that; $z_1 = 3 + 4i$ and $z_2 = 1 - 2i$, find (a) $z_1 + z_2$, (b) $z_1 - z_2$, and (c) $z_1 z_2$ (4 marks)

e) Find the roots of the cubic equation $x^3 + 3x^2 - x - 3 = 0$ and sketch a graph of $y = x^3 + 3x^2 - x - 3$. (6 marks)

Question Three

a) A particle moving in the xy plane undergoes a displacement given by $\Delta\vec{r} = (2.0\hat{i} + 3.0\hat{j})m$ as a constant force $\vec{F} = (5.0\hat{i} + 2.0\hat{j})N$ acts on the particle. Calculate the work done by $\vec{F}$ on the particle. (3 marks)

b) **(i)** Find $\int (2x^3 - 3x + 4)dx$ (2 marks)

(ii) Evaluate $\int_0^{\frac{3}{2}} \frac{dx}{\sqrt{9 - x^2}}$. (3 marks)

c) **(i)** Find the gradient of the scalar field; $f(x, y, z) = xy^2 - yz$ (3 mark)

(ii) Find the divergence of the vector field $F = 3x^2\hat{i} - 6xy\hat{j}$. Explain the result (2 marks)

(iii) Find the curl of the vector field $F = y^3\hat{i} + xy\hat{j} - z\hat{k}$. (3 marks)

d) Determine the probabilities of selecting at random (i) a man, and (ii) a woman from a crowd containing 20 men and 33 women. (4 marks)

Question Four

a) (i) Write down the polar form of a complex number**.** (1 marks)

(ii) Find $z_1 z_2$ if $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $z_2 = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$. (3 marks)

b) The functions are defined by:

$$f(x) = 2x \text{ for } x \in \Re$$
$$g(x) = x^2 \text{ for } x \in \Re$$
$$h(x) = \frac{1}{x} \text{ for } x \in \Re, x \neq 0.$$

Find the following.

(i) $fg(x)$ (ii) $gf(x)$ (iii) $gh(x)$ (iv) $f^2(x)$ (v) $\mathrm{fgh}(x)$ (5 marks)

c) Find all the stationary points on the curve of $y = 2t^4 - t^2 + 1$ and sketch the curve.

(6 marks)

d) Find $\left(\sqrt{3} + i\right)^{10}$ in the form $a + ib$ (5 marks)

Question Five

a) A force of $\vec{F} = (2.0\hat{i} + 3.0\hat{j})N$ is applied to an object that is pivoted about a fixed axis aligned along the z-coordinate axis. The force is applied at a point located at $\vec{r} = \left(4.0\hat{i} + 5.0\hat{j}\right)m$. Find the torque $\vec{\tau}$ applied to the object. (4 marks)

b) Find $z_1 z_2$ if $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $z_2 = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$. (4 marks)

c) Find the roots of the cubic equation $x^3 + 4x^2 + x - 26 = 0$ (4 marks)

d) Write the mathematical expression for de Moivre's theorem. Using the theorem, simplify $\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3$. (3 marks)

e) **(i)** Find $f^{-1}(x)$ when $f(x) = x^2 + 2, x \geq 0$ (3 marks)

(ii) Find $f(7)$ and $f^{-1}f(7)$. What do you notice? (2 marks)