MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR BACHELOR OF EDUCATION (SCIENCE)

SPH 300: WAVE THEORY

DATE: 26/9/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Important Constants

Speed of light in vacuum $= 3.0 \times 10^8$ m/s

Refractive index of glass = 1.5

QUESTION ONE (30 marks)

a) (i) Distinguish between longitudinal and transverse waves giving an example in each case. (3 marks)

(ii) The equation for a transverse wave in a stretched string is $y = a \sin 2\pi \left(\frac{t}{0.04} - \frac{x}{40}\right)$ If the lengths are in cm and time in seconds, calculate the wavelength, frequency and amplitude of the wave. (3 marks)

b) (i) State any three characteristics of a body executing simple harmonic motion. (3 marks)

(ii) The equation of motion of a simple harmonic oscillator is given as $\frac{d^2x}{dt^2} + \omega^2 x = 0$ where x is the displacement and ω is the angular velocity. Show that the following relations satisfy the wave equation and sketch the displacement in a phasor diagram.

1. $x = A \sin \omega t$ (3 marks)
2. $x = A \cos \omega t$ (3 marks)

c) A mass-spring system having mass m and springs constant k is executing simple harmonic motion about a fixed point. Derive the expression of the periodic time for the system. (6 marks)

d) Figure 1 shows a periodic triangular wave of a rectifier circuit. Use the trigonometric Fourier series analysis approach to calculate the first three Fourier coefficients of the waveform, (6 marks)

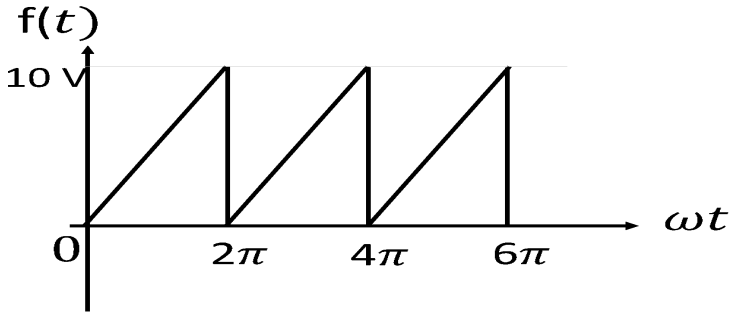

Figure 1

e) Explain with the aid of a suitable diagram, the difference between under damping, over damping and critical damping of motion oscillators. (3 marks)

QUESTION TWO (20 MARKS)

a) (i) Describe the conditions necessary for an observable interference pattern to occur.
(3 marks)

(ii) In double slit experiment interference, why is the separation (D) between the slits and the screen supposed to be far much longer than the separation between the slits (d)?
(2 marks)

b) (i) Determine the linear separation between the two adjacent maxima (Δy) in an interference pattern for small angular separation θ.
(7 marks)

(ii) Given that the wavelength of light $\lambda = 10\ \mu$m, distance between the slit and the screen, D = 30 cm and width of the slit, d = 1.5 mm in a diffraction experiment, calculate the linear separation between two adjacent maxima.
(2 marks)

c) (i) Distinguish between Fresnel and Fraunhofer diffraction.
(2 marks)

(ii) With the aid of a suitable diagram, show how Fraunhofer conditions can be attained in the laboratory.
(4 marks)

QUESTION THREE (20 marks)

a) (i) A thin film of liquid (e.g. soap) appears bright or dark when viewed in a monochromatic light. Explain this observation. You may be awarded a mark for clarity of your answer.
(3 marks)

(ii)An air wedge is formed between two glass plates which are in contact at one end and separated by a piece of wire at the other end. Calculate the diameter of the wire if 30 dark fringes are observed between the ends when light of wavelength 6×10^{-7} m is incident normally on the wedge. (5 marks)

b) Determine the Fourier series of the periodic triangular waveform shown in Figure 2.
(8 marks)

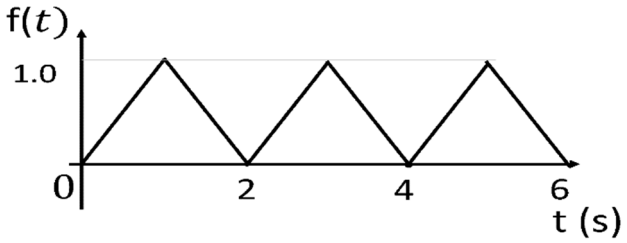

Figure 2

c) Two harmonic oscillations of frequency ω_0 having amplitude 1.0 cm and initial phases 0 and $\pi/2$, respectively, are superposed. Calculate the amplitude and the phase of the resultant vibration. (4 marks)

a) (i) Distinguish between polarization and diffraction of waves (2 marks)

(ii)Using the idea of polarization of light by reflection at an interface, deduce Brewster's law (4 marks)

b) If a plate of glass is used as a polarizer , find,

i. The polarizing angle. (3 marks)

ii. The angle of refraction (2 marks)

c) (i)Red light of wavelength 6500 Å from a distant source falls on a slit 0.5 mm wide. What is the distance between the two dark bands on either side of the central bright band of the diffraction pattern observed on a screen 1.8 m from the slit?

(3 marks)

(ii)Why does the intensity of secondary maxima go on decreasing with distance from the central maximum?

(3 marks)

d) What is the difference in diffraction pattern at a single slit due to monochromatic and white light? (3 marks)

QUESTION FIVE (20 MARKS)

a) Represent the following wave disturbances as phasor diagrams

i. $E_1 = E_o \sin \omega t$ (2 marks)

ii. $E_2 = E_o \sin\left(\omega t + \frac{\pi}{2}\right)$ (2 marks)

iii. $E = E_1 + E_2$ where E_1 and E_2 are as in (i) and (ii) (4 marks)

b) (i) Distinguish between a forced oscillation and a free oscillation.

(2 marks)

(ii) For harmonic wave, the displacement of a particle at a distance x from the centre of disturbance is given by $y = a \sin \frac{2\pi}{\lambda}(vt - x)$ where a is amplitude of the wave and v is the phse velocity. Deduce the wave equation $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ (5 marks)

c) Consider the following simple harmonic oscillations

$$x_1 = a_1 \cos(\omega_1 t)$$
$$x_2 = a_2 \cos(\omega_2 t)$$

Use Complex number analysis to obtain the following expression of the amplitude for the resultant motion:

$$a = [a^2 + a^2 + 2a_1 a_2 \cos(\omega_1 - \omega_2)t]^{1/2}$$

Show that the resultant amplitude oscillates between the values $a_1 + a_2$ and $a_1 - a_2$.

(5 marks)