



MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR
BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION
TECHNOLOGY)

BACHELOR OF SCIENCE (MATHEMATICS AND STATISTICS)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ARTS

SMA 104: CALCULUS 1

DATE: 13uj/4/2023
PM

TIME: 2:00 – 4:00

Instructions to Candidates.

Answer question ONE and any other TWO questions.

QUESTION ONE: 30 MARKS (COMPULSORY)

- a) Evaluate;
- Use squeeze theorem to find $\lim_{n \rightarrow 0} \frac{\sin^2 x}{x^2}$ (3 marks)
 - $\lim_{x \rightarrow -\infty} \left(\frac{3x^5 + x^2 - 5}{6 + x + 7x^2} \right)$ (3 marks)
 - $\frac{dy}{dt}$ when $y = \frac{\sinh t}{t+1}$ (3 marks)
- b) Investigate the continuity of the function below at $x = 1$ (3 marks)
- $$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

- c) Find derivative of $f(x) = \sqrt{x+1}$ using definition (5 marks)
- d) Find $\frac{dy}{dx}$ given that $x = 3 \cos t, y = 3 \sin t$ (4 marks)
- e) By differentiating the equation $x^2 - xy - y^2 = 3$ implicitly, find $\frac{dy}{dx}$ (3 marks)
- f) Find the equation of the normal line to $x^2 - y^2 = 7$ at the point $(4, -3)$ (3 marks)
- g) Given $h(x) = \frac{x+4}{2x-5}$, find $h^{-1}(x)$ (3 marks)

QUESTION TWO (20 MARKS)

- a) Find the derivative of $y = \cos x$ from first principles (5 marks)
- b) Let $y = \frac{2x}{\sqrt{3x^2+4}}$, show that $\frac{dy}{dx} = \frac{8}{(3x^2+4)^{3/2}}$ (5 marks)
- c) Find $\frac{d^2y}{dx^2}$, if $x = 3t^2 + 1, y = t^3 - 2t^2$ (5 marks)
- d) Find $\frac{dy}{dx}$ given that $x^2 \tan y + y^5 \sec x = 2x$ (5 marks)

QUESTION THREE (20 MARKS)

- a) Differentiate the following with respect to x
- $y = x^2 \sin^{-1} x$ (4 marks)
 - $y^2 + 6x = x^3$ (3 marks)
 - $y = (2x^2 + 1)^{\frac{1}{2}}(x + 3)$ (3 marks)
 - $y = \sec x$ (4 marks)
- b) Evaluate $\lim_{x \rightarrow 0} \frac{x}{\sin 7x}$ (3 marks)
- c) Find both vertical and horizontal asymptotes of $y = \frac{2x}{x+1}$ (3 marks)

QUESTION FOUR (20 MARKS)

- a) Find the critical points of $g(t) = \sqrt[3]{t^2}(2t - 1)$ (5 marks)
- b) The position of a cheetah moving across a level ground in a straight line chasing after prey is given by the equation $s(t) = t^3 - 15t^2 + 63t$, where t is measured in second and s in metres, determine;
- Time when the cheetah was momentarily at rest (3 marks)
 - Position of the cheetah when velocity was zero (3 marks)
 - Acceleration of the cheetah at $t = 2.5$ s (2 marks)
- c) Evaluate the following;
- $\lim_{x \rightarrow \infty} \frac{x^2}{2^x}$ (3 marks)

ii. $\frac{dy}{dx}$ given $y = \tan^{-1}(x)$ (4 marks)

QUESTION FIVE (20 MARKS)

- a) Sketch the curve $y = 3x^4 - 8x^3 + 6x^2 - 3$ (10 marks)
- b) Air is being pumped into a spherical balloon at a rate of $45\text{cm}^3/\text{minute}$. Find the rate of change of radius when the radius is 2 cm . (5 marks)
- c) Find the linear approximation to $f(x) = \cos x$ at $x_0 = \frac{\pi}{3}$ and use it to approximate $\cos(1)$ (5 marks)