MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

ECU 301: ENGINEERING MATHEMATICS X

Date: …….. **Time: ……**

Instructions to Candidates

Answer question ONE and any other TWO questions

QUESTION 1: 30 MARKS (COMPULSORY)

a) Prove each of the following

i) $\nabla\Delta = \Delta - \nabla$ (4 marks)

ii) $\mu^2 f_i = \frac{1}{4}(f_{i+1} + 2f_i + f_{i-1})$ (3 marks)

b) Derive Newton's Forward Difference Interpolating Polynomial of degree n (4 marks)

c) Evaluate the integral below using Trapezoidal rule with 4 and 8 subdivisions

$\int_0^2 \frac{1}{1+x} dx$ (6 marks)

d) Show that the relative error in a product is equal to the the sum of the relative errors

$(V_{xy} = V_x + V_y)$ (3 marks)

e) Using Lagrange's quadratic interpolating polynomial find the value of f(x) corresponding to x=2.3

x	1.1	1.7	3.0
f(x)	10.6	15.2	20.3

(5 marks)

f) Use linear interpolation to estimate the value of velocity at t=2.6 given the data below

t (s)	1	2	3
v(t) (m/s)	2.03	4.23	6.84

(5 marks)

QUESTION 2 (20 MARKS)

a) Evaluate $\int_0^1 \frac{1}{1+x} dx$ using Gaussian three point integration rule. Find the exact value and the error in the calculation in 6 d.p (10 marks)

b) Given the data

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	-3	-65	-49	-15	1	-13	-45	-69	5

i) Locate the error in the data given and obtain a corrected finite difference table (6 marks)

ii) Extend the table to obtain the value of f(5) (4 marks)

QUESTION 3 (20 MARKS)

a) Use Newton-Ralphson iterative formula to get a root of the equation given below

$x^3 - 3x + 1 = 0$ Near x=2 to 4 d.p (8 marks)

b) The profits of a company are as given in the table below (in thousands)

Year (x)	2000	2003	2006	2009	2012
Profit y=f(x)	120	100	111	108	99

i) Calculate the profits for the year 2001 (f(2001)) using Newton Forward difference polynomial (6 marks)

ii) Calculate the rate of change of profit for the year 2001 ($(f'(2001))$ using Newton Forward difference polynomial (6 marks)

QUESTION 4 (20 MARKS)

a) Derive the linear interpolating polynomial (4 marks)

b) Use the above (in a) derivation to estimate the value of f(2.05) give the data in the table below;

x	2.0	2.1	2.2	2.3	2.4
f(x)	2.218	2.628	3.058	3.508	3.978

(6 marks)

c) Use Taylor series method to find the solution of the initial value problem given as

$\frac{dy}{dt} = t + y$; y(1)=0 at t=1.2, h=0.1 (10 marks)

QUESTIONS 5 (20 MARKS)

a) Given the initial value problem below, find y approximately at x=0.1 in 5 steps using Euler's method

$\frac{dy}{dt} = \frac{y-t}{y+t}$; y(0)=1 (8 marks)

b) Solve the following differential equation using 4^{th} order Runge-Kutta method from t=0 to t=0.2 taking h=0.1 (12 marks)