MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION (SCIENCE)

SMA 466: MULTIVARIATE STATISTICAL METHODS II

DATE: **TIME:**

INSTRUCTIONS: Answer **question one** and **any other two** questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Let **Y** be distributed as $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$,where, $\boldsymbol{\mu} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, $\boldsymbol{\Sigma} = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$, Find the conditional distribution of y_2 given y_1 and y_3. (5 marks)

b) Find the distribution of; $\mathbf{Y} = \begin{pmatrix} 2x_1 - x_2 + x_3 \\ -2x_2 + 3x_3 \end{pmatrix}$,given that; $\mathbf{X} \sim N_3\left[\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 4 & 1 & 2 \\ 1 & 9 & 7 \\ 2 & 7 & 10 \end{pmatrix}\right]$. (5 marks)

c) Suppose that;

$$\boldsymbol{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \sim N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \text{where } \boldsymbol{\mu} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 7 \end{pmatrix}$$

Let $\boldsymbol{X_1} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\boldsymbol{X_2} = x_3$ Find the marginal distribution of $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ (5 marks)

d) Consider the intelligence scores data on 101 subjects where $\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} = \begin{pmatrix} 55.24 \\ 34.97 \end{pmatrix}$,
$\boldsymbol{S_U} = \begin{pmatrix} 210.54 & 126.99 \\ 126{,}99 & 119.68 \end{pmatrix}$,construct 99% simultaneous confidence interval for μ_1 *and* μ_2 (5 marks)

e) For a bivariate normal distribution, use the data below to test at $\alpha = 0.05$ level the hypothesis,

$H_0; \boldsymbol{\mu}=(3.4 \quad 6)'$ versus $H_1; \boldsymbol{\mu} \neq (3.4 \quad 6)'$ (5 marks)

$$\boldsymbol{X} = \begin{pmatrix} 3 & 4 & 5 & 6 & 2 \\ 9 & 5 & 7 & 2 & 8 \end{pmatrix}$$

a) In an investigation of adult intelligence, scores were obtained on two tests "verbal" and "performance" for 101 subjects aged 60 to 64.the reported mean score and covariance matrix were; $\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix}=\begin{pmatrix} 55.24 \\ 34.97 \end{pmatrix}$, $\boldsymbol{S_U}=\begin{pmatrix} 210.54 & 126.99 \\ 126{,}99 & 119.68 \end{pmatrix}$,at 0.01 level test the hypothesis that;$\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}=\begin{pmatrix} 60 \\ 50 \end{pmatrix}$

(5 marks)

QUESTION TWO (20 MARKS)

a) Some students did tests in three units and the results were as follows;

Test 1: 17 30 17 22 16
Test 2: 29 25 30 28 30
Test 3: 17 23 24 18 28

Compute the;

i. Mean vector. (2 marks)
ii. Sample covariance. (4 marks)
iii. Correlation matrix. (4 marks)

b) A laboratory analysis of two different nutrients (A and B) for each of a sample of size n=10of the same food (in mg per 100gm portion) revealed the following;

A	3.17	3.45	3.73	1.82	4.39	2.91	3.54	4.09	2.85	2.05
B	3.45	2.35	5.09	3.88	3.64	4.63	2.88	3.98	3.74	4.36

i. Does it appear that the sample come from a food with mean nutrient amount vector $\mu_0 = (3{,}5)'$ (5 marks)
ii. Find the 95% confidence interval for mean nutrient A and B. (5 marks)

QUESTION THREE (20 MARKS)

a) Let X$\sim N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ be a tri-variate normal random vector. Suppose a certain sample gave

$$S=\begin{pmatrix} 64 & 0 & 8 \\ 0 & 4 & 0 \\ 8 & 0 & 16 \end{pmatrix}$$,Find the ;

i. Eigen values of this matrix. (3 marks)
ii. The first two principal Components, (5 marks)
iii. Total variance explained by the first two components. (2 marks)

b) Given the following hypothetical data on academic performance of students (in high school out of 100 and in university out of 4.00), Test the equality of the population mean vectors between the two groups. (10 marks)

Female		
97	95	85
3.40	3.45	3.50

Male				
86	84	70	80	75
3.90	3.75	2.25	3.05	2.80

QUESTION FOUR (20 MARKS)

Given that X~ $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with,

$$\boldsymbol{\mu} = \begin{pmatrix} 44 \\ 20 \\ 16 \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} 64 & 8 & -16/5 \\ 8 & 16 & 4 \\ -16/5 & 4 & 4 \end{pmatrix},$$

Find;

a) The distribution of $\mathbf{Y}=\begin{pmatrix} x_1 - x_3 \\ x_1+2x_2 + x_3 \end{pmatrix}$. (5 marks)

b) The correlation matrix of the data. (3 marks)

c) The distribution of X_1 given $X_2 = 15$ and $X_3 = 18$. (5 marks)

d) The regression function of X_3 on X_1 and X_2. (4 marks)

e) The partial correlation coefficient between X_1 and X_3 for fixed values of X_2. (3 marks)

QUESTION FIVE (20 MARKS)

a) State the four assumptions MANOVA makes about the data (4 marks)

b) observations on three responses are collected for three treatments.

The observation vectors; $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ are as given below

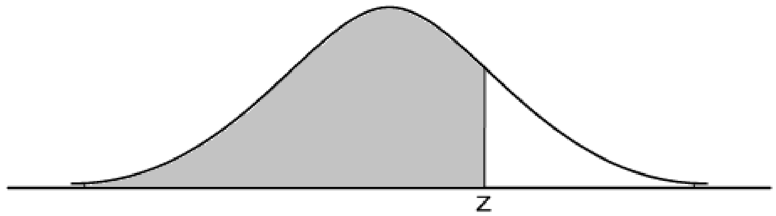
Tmt 1; $\begin{pmatrix} 14 \\ 17 \\ 16 \end{pmatrix}, \begin{pmatrix} 16 \\ 11 \\ 18 \end{pmatrix}$,Tmt 2; $\begin{pmatrix} 18 \\ 16 \\ 16 \end{pmatrix}, \begin{pmatrix} 14 \\ 16 \\ 15 \end{pmatrix}$, Tmt 3; $\begin{pmatrix} 20 \\ 18 \\ 16 \end{pmatrix}, \begin{pmatrix} 18 \\ 14 \\ 14 \end{pmatrix}. \begin{pmatrix} 16 \\ 10 \\ 15 \end{pmatrix},$

Find;

i. The matrix of sum of squares due to treatment. (6 marks)

ii. The matrix of residual sum of squares. (6 marks)

iii. The wilk's lambda statistic and use it to test the hypothesis that there is no treatment effects.(use $\alpha = 0.05$). (4 marks)



	Area under the standard normal curve from 0 to Z									
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000