MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (PROGRAMMING AND STATISTICS)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF SCIENCE (ACTUAL SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

SMA 202: LINEAR ALGEBRA I

DATE: 9/12/2021 **TIME:**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Considering mathematical context, define the following vector space terms;

i. Vector subspace (1 mark)

ii. Spanning sets (1 mark)

iii. Linear dependence of vectors (1 mark)

iv. Basis (1 mark)

b) Use the Cramer's rule to solve the systems of equations;

$$2x+3y=1$$
$$5x+7y=3$$
(4 marks)

c) A stable floor model of an aircraft is determined by the points $A(2,\ -1,\ 1),\quad B(3,\ 2,\ -1),\quad C(-1,\ 3,\ 2)$. Calculate its equation. (4 marks)

d) Show that the following vectors are coplanar;

$$\vec{a}=4i-j+k$$
$$\vec{b}=3i-2j-k$$
$$\vec{c}=i+j+2k$$
(4 marks)

e) Determine the vector projection of the point $A(2,\ 7)$ *on the point* $B(-3,\ 1)$ (4 marks)

f) Consider the matrix;

$$S=\begin{pmatrix}10 & 7 & 8 & 7\\ 7 & 5 & 6 & 5\\ 8 & 6 & 10 & 9\\ 7 & 5 & 9 & 10\end{pmatrix}$$

Reduce S to its Echelon form (5 marks)

g) Consider the matrix

$$A=\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 1 & 5 & 7\end{pmatrix}$$

Calculate A^{-1} (5 marks)

QUESTION TWO (20MARKS)

a) Given that

$$\vec{a}=a_1i+a_2j+a_3k$$
$$\vec{b}=b_1i+b_2j+b_3k$$
$$\vec{c}=c_1i+c_2j+c_3k$$

Calculate;

i. $(\vec{a}\times\vec{c})+2\vec{b}$ (5 marks)

ii. $(2\vec{a}\times\vec{b})\times\vec{c}$ (7 marks)

b) Solve the following system of equations using the matrix inversion method (8 marks)

$$x_1 + 2x_2 + x_3 = 4$$
$$-2x_3 - 3x_1 - 4x_2 = 2$$
$$3x_2 + 5x_3 + 5x_1 = -2$$

QUESTION THREE (20 MARKS)

a) Consider the parallelogram whose adjacent sides are given by

$2i - 4j + 5k$ *and* $i - 2j - 3k$. Calculate;

i. Unit vector parallel to its diagonal (4 marks)

ii. Area of the parallelogram (5 marks)

b) Using reduction to echelon form solve

$$2w + x + y - 2z = 10$$
$$2y + 4w + z = 8$$
$$3w + 2x + 2y = 7$$
$$w + 3x + 2y - z = -5$$

(11 marks)

QUESTION FOUR (20 MARKS)

a) Determine the parametric equation of a line whose Cartesian equation is given by;

$$\frac{x-1}{5} = \frac{y+1}{2} = \frac{z}{-5}$$

(4 marks)

b) A ship construction company describes a ship's floor and inclined wall model using the equations given by $2x + 3y - 4z + 1 = 0$ *and* $3x - 5y + 6z + 8 = 0$ respectively. Calculate the angle between the floor and its inclined wall model. (6 marks)

c) Prove that the equation of a plane passing through any three non-collinear points $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$ is given by;

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

(10 marks)

QUESTION FIVE (20 MARKS)

a) In the context of matrices concept. Define the following;

i. Symmetric matrix (3 marks)

ii. Skew-symmetric matrix (3 marks)

b) Show that; $A(3, 0, -3)$, $B(-1, 1, 2)$, $C(4, 2, -2)$, $D(2, 1, 1)$ are linearly dependent (6 marks)

c) Let V be a subspace of $\Re^4$ spanned by $(1, 0, 2, 1)$, $(1, -1, 1, 2)$, $(0, 0, 3, 1)$

Determine the Basis and Dimension of V (8 marks)