



# MACHAKOS UNIVERSITY

University Examination 2018/2019

**SCHOOL OF ENGINEERING AND TECHNOLOGY**  
**DEPARTMENT OF BUILDING AND CIVIL ENGINEERING**  
**THIRD YEAR SECOND SEMESTER EXAMINATION FOR**  
**BACHELOR OF SCIENCE (CIVIL ENGINEERING)**  
**ECV 312: THEORY OF STRUCTURES IV**

**DATE: 20/5/2019**

**TIME: 11.00-1.00 PM**

**INSTRUCTIONS:** Attempt questions **ONE** and any other **TWO** questions.

**QUESTION ONE (COMPULSORY) (30 MARKS)**

Determine the reactions and the force in each member of the truss shown in the Figure 1 below by finite element method.  $EA = \text{constant}$

**QUESTION TWO (20 MARKS)**

Determine the reactions and the member end forces for the three-span continuous beam shown in the Figure 2 below using finite element method.  $E = 200 \text{ GPa}$ ,  $I = 216(10^6) \text{ mm}^4$

**QUESTION THREE (20 MARKS)**

Determine the reactions and the member forces the two-member frame shown in the Figure 3 below. Take  $I = 243(10^6) \text{ mm}^4$ ,  $A = 6000 \text{ mm}^2$  and  $E = 200 \text{ GPa}$  for both members.

**QUESTION FOUR (20 MARKS)**

Determine the reactions and the member end forces of the frame shown in the Figure 4 below. Take  $I = 250(10^6) \text{ mm}^4$ ,  $A = 900 \text{ mm}^2$  and  $E = 200 \text{ GPa}$  for each member.

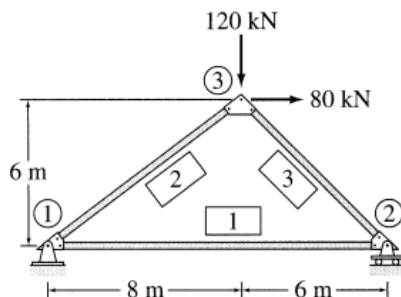


Figure 1

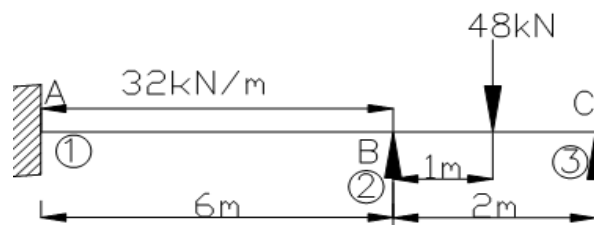


Figure 2

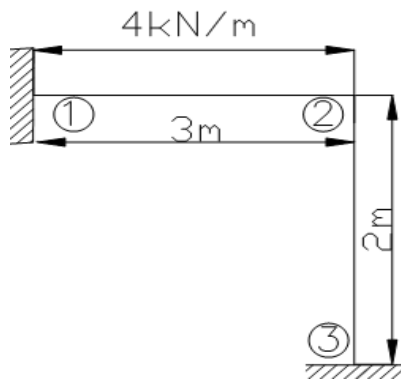


Figure 3

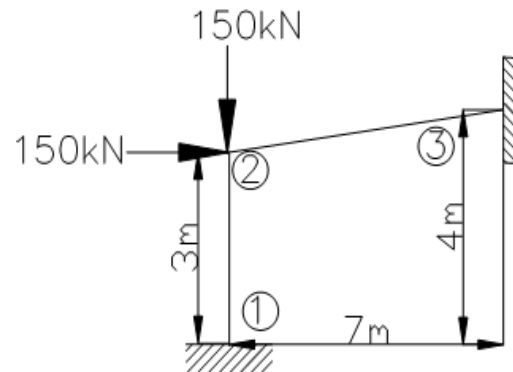


Figure 4

### HINTS

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$\lambda_x = \cos \theta_x$$

$$\lambda_y = \cos \theta_y$$

$$\begin{pmatrix}
\left(\frac{AE}{L} \lambda_{ix}^2 + \frac{12EI}{L^3} \lambda_{iy}^2\right) & \left(\frac{AE}{L} - \frac{12EI}{L^3}\right) \lambda_{ix} \lambda_{iy} & -\frac{6EI}{L^2} \lambda_{iy} & -\left(\frac{AE}{L} \lambda_{ix} \lambda_{jx} + \frac{12EI}{L^3} \lambda_{iy} \lambda_{jy}\right) & -\left(\frac{AE}{L} \lambda_{ix} \lambda_{jy} - \frac{12EI}{L^3} \lambda_{iy} \lambda_{jx}\right) & -\frac{6EI}{L^2} \lambda_{iy} \\
\left(\frac{AE}{L} - \frac{12EI}{L^3}\right) \lambda_{ix} \lambda_{iy} & \left(\frac{AE}{L} \lambda_{iy}^2 + \frac{12EI}{L^3} \lambda_{ix}^2\right) & \frac{6EI}{L^2} \lambda_{ix} & -\left(\frac{AE}{L} \lambda_{iy} \lambda_{jx} - \frac{12EI}{L^3} \lambda_{ix} \lambda_{jy}\right) & -\left(\frac{AE}{L} \lambda_{iy} \lambda_{jy} + \frac{12EI}{L^3} \lambda_{ix} \lambda_{jx}\right) & \frac{6EI}{L^2} \lambda_{ix} \\
-\frac{6EI}{L^2} \lambda_{iy} & \frac{6EI}{L^2} \lambda_{ix} & \frac{4EI}{L} & \frac{6EI}{L^2} \lambda_{jy} & -\frac{6EI}{L^2} \lambda_{jx} & \frac{2EI}{L} \\
-\left(\frac{AE}{L} \lambda_{ix} \lambda_{jx} + \frac{12EI}{L^3} \lambda_{iy} \lambda_{jy}\right) & -\left(\frac{AE}{L} \lambda_{iy} \lambda_{jx} - \frac{12EI}{L^3} \lambda_{ix} \lambda_{jy}\right) & \frac{6EI}{L^2} \lambda_{jy} & \left(\frac{AE}{L} \lambda_{jx}^2 + \frac{12EI}{L^3} \lambda_{jy}^2\right) & \left(\frac{AE}{L} - \frac{12EI}{L^3}\right) \lambda_{jx} \lambda_{jy} & \frac{6EI}{L^2} \lambda_{jy} \\
-\left(\frac{AE}{L} \lambda_{ix} \lambda_{jy} - \frac{12EI}{L^3} \lambda_{iy} \lambda_{jx}\right) & -\left(\frac{AE}{L} \lambda_{iy} \lambda_{jy} + \frac{12EI}{L^3} \lambda_{ix} \lambda_{jx}\right) & -\frac{6EI}{L^2} \lambda_{jx} & \left(\frac{AE}{L} - \frac{12EI}{L^3}\right) \lambda_{jx} \lambda_{jy} & \left(\frac{AE}{L} \lambda_{jy}^2 + \frac{12EI}{L^3} \lambda_{jx}^2\right) & -\frac{6EI}{L^2} \lambda_{jx} \\
-\frac{6EI}{L^2} \lambda_{iy} & \frac{6EI}{L^2} \lambda_{ix} & \frac{2EI}{L} & \frac{6EI}{L^2} \lambda_{jy} & -\frac{6EI}{L^2} \lambda_{jx} & \frac{4EI}{L}
\end{pmatrix}$$

### FIXED END FORCES

At A

$$M_A = -\frac{Wab^2}{L^2}$$

$$R_A = -\frac{Wb}{L^3}(l^2 + ab - a^2)$$

At B

$$M_B = -\frac{Wa^2b}{L^2}$$

$$R_A = -\frac{Wa}{L^3}(l^2 + ab - b^2)$$

