MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (CLOUD COMPUTING)

SCO 111/SIT 192: CALCULUS I

DATE: 11/4/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer Question One and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine $f(g(x))$ and $g(f(x))$ given that $f(x) = \frac{2x+3}{x-1}$, $g(x) = \frac{x+3}{x-2}$ (4 marks)

b) Use the first principles to determine the derivative $\frac{dy}{dx}$ given $y = \frac{3x+4}{2x-3}$ (5 marks)

c) Evaluate the following;

i) $\lim\limits_{x\to\infty} \frac{3x^2-13x-10}{2x^2-7x-15}$ (3 marks)

ii) $\lim\limits_{x\to1} \frac{x^2+4x-5}{x^2-1}$ (3 marks)

d) Given that $x + xy + y = 2$determine $\frac{dy}{dx}$. Hence find the equation of tangent line to the graph of the given function at point P (1, 2). (4 marks)

e) The total sales s (in thousands) for a home video game t months after it was introduced is given by $s(t) = \frac{250t^2}{t^2+100}$. Find $s'(0)$ and interpret your results (4 marks)

f) Determine the derivative of the following function using chain and quotient rules; (4 marks)

$$f(x) = \left[\frac{x^2}{x+1}\right]^5$$

g) Use product rule to differentiate $h(x) = e^{2x^3}(x^2 + x + 2)$ (3 marks)

QUESTION TWO (20 MARKS)

a) In a medical treatment 500 milligrams of a drug are administered to a patient. At time t hours after the drug is administered X milligrams of the drug remain in the patient. The doctor has a mathematical model which states that $\mathbf{X = 500e^{-2t}}$ Find;

i) The derivative of X with respect to t (2 marks)

ii) The quantity X and rate of change of X at t = 5. Hence using your result estimate X when t = 6. (3 marks)

b) Use the first principles to show that $\frac{d}{dx}(\cos x) = -\sin x$ (5 marks)

c) A mobile phone manufacturer has introduced a new phone in the market. The marketing manager determines that t weeks after an advertising campaign begins, P(t) percent of the potential market is aware of the new phone, where $P(t) = \frac{100(t^2+5t+5)}{t^2+10t+30}$;

i) Determine the rate of change of percentage P(t) at 5th week. (4 marks)

ii) What happens to the P(t) and the rate of change of P(t) in the long term?

(6 marks)

QUESTION THREE (20 MARKS)

a) An environmental study of a certain suburban community suggests that the average daily level of carbon monoxide in the air will be $\mathbf{c(p)} = \sqrt{\mathbf{0.5p^3}}$ parts per million when the population is p thousand. It is estimated that t years from now, the population of the community will be $\mathbf{p(t) = 3.1 + 0.1t^2}$ thousand. What is the rate of change of carbon monoxide level at 3 years from now? (9 marks)

b) An epidemiologist determines that a particular epidemic spreads in such a way that t weeks after the outbreak, N hundred new cases will be reported, where $N(t) = \frac{5t}{12+t^2}$;

i) Find $\frac{dN}{dt}$ and $\frac{d^2N}{dt^2}$ (6 marks)

ii) At what time is the epidemic at its worst? What is the maximum number of reported new cases? (5 marks)

QUESTION FOUR (20 MARKS)

a) Obtain all the critical numbers of $f(x) = -2x^3 + 3x^2 + 12x - 5$ and use the second derivative test to classify each of them as either relative maxima or minima. Then sketch the graph of $f(x)$. (10 marks)

b) Use the first principle to get the derivative $\frac{dy}{dx}$ for $y = \sqrt{2x+3}$ (4 marks)

c) A manufacturer determines that x units of a particular luxury item will be sold when the price is $p(x) = 12 - x \ \ln x^3$ hundred dollars per unit;

i) If the revenue is $\boldsymbol{R(x) = xp(x)}$ obtain the rate of change of revenue. (3 marks)

ii) Estimate the revenue obtained from producing the 5th unit. What is the actual revenue obtained from producing the 5th unit? (3 marks)

QUESTION FIVE (20 MARKS)

a) Sketch the graph of a function f(x) that has all of the following properties:

i) Discontinuities at $\boldsymbol{x = -1}$ and at $\boldsymbol{x = 3}$

ii) $\boldsymbol{f'(x) > 0}$, for $\boldsymbol{x < 1, x \neq -1}$

iii) $\boldsymbol{f'(x) < 0}$, for $\boldsymbol{x > 1, x \neq 3}$

iv) $\boldsymbol{f''(x) > 0}$, for $\boldsymbol{x < 1, x > 3}$ and $\boldsymbol{f''(x) < 0}$, for $\boldsymbol{-1 < x < 3}$

v) $\boldsymbol{f(0) = 0 = f(2), f(1) = 3}$ (9 marks)

b) Find the second derivative of the given function

i) $\boldsymbol{y = \frac{x^2}{1+2x}}$ (5 marks)

ii) $\boldsymbol{y = \ln\sqrt{t^2+1}}$ (6 marks)