



MACHAKOS UNIVERSITY

University Supplementary Examinations

**BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE,
BACHELOR OF EDUCATION, BACHELOR OF MATHEMATICS**

SMA 431. DIFFERENTIAL GEOMETRY

**INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO
QUESTIONS**

QUESTION ONE (30 MARKS)

- a. Two vectors **A** and **B** are given by $\mathbf{A} = 6\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$
- (i) Compute the scalar and vector products of the two vectors (5 Marks)
 - (ii) Find the angle between **A** and **B**. (2 Marks)
 - (iii) Determine a unit vector perpendicular to the plane of **A** and **B**. (3 Marks)
- b. A space curve is defined by $\mathbf{r} = 2\mathbf{i} \sin 3t + 2\mathbf{j} \cos 3t + 8\mathbf{k}t$.
- Determine velocity and acceleration of a particle moving along the curve. (2 marks)
- Find the magnitude of the velocity and acceleration. (2 marks)
- c. A curve is defined by the vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + 2\mathbf{k}$ where $x = 1 + t^2$, $y = 4t - 3$ and $z = 2t^2 - 6t$.
- i) Determine the tangent and unit tangent to the curve. (3 marks)
 - ii) Determine the curvature and radius of curvature at any point on the curve. (4 marks)
- d. Determine the length of the arc of the curve $\mathbf{r} = e^t \cos t \mathbf{i} + e^t \sin t \mathbf{j} + e^t \mathbf{k}$ from $t = 0$ and $t = \pi$. (5 marks)

- e. Find the curvature and torsion for the space curve $x = t$, $y = t^2$ and $z = t^3$ called the twisted cubic. (4 marks)

QUESTION TWO (20 MARKS)

- a. List any three types of planes of a space curve giving the equation for each. (6 marks)
- b. Find a unit normal to the surface $\phi(x, y, z) = xy - z^2 = 0$ at the point $P(1, 4, -2)$. (3 marks)
- c. Find the equation of the circle of curvature of $2xy + x + y = 4$ at the point $(1, 1)$. (11 marks)

QUESTION THREE (20 MARKS)

- a. Consider the surface $\mathbf{r} = u\mathbf{i} + v\mathbf{j} + (uv - v)\mathbf{k}$. Calculate the first quadratic differential form in general and also when $u = -1$ and $v = 1$ (5 marks)
- b. Given that $f(x) = \sqrt{1-x}$ on $\left[0, \frac{1}{2}\right]$, find the surface area of the curve rotated on the x axis (7 marks)
- c. Find the $\frac{dS}{dx}$ and $\frac{dS}{dy}$ on the ellipse $x^2 + 4y^2 = 8$ (8 marks)

QUESTION FOUR (20 MARKS)

- a. Find the fundamental magnitudes of the first and the second order of the surface; $\mathbf{r} = u\mathbf{i} + 2v\mathbf{j} + 2uv\mathbf{k}$ (10 marks)
- b. Find the curvature of the parabola $y^2 = 12x$ at the points $(3, 6)$ and $(0, 0)$ (10 marks)

QUESTION FIVE (20 MARKS)

- a. Compute the normal and the geodesic curvature of the circle $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$ on the elliptic parabolic $\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + (u^2 + v^2)\mathbf{k}$ (10 marks)
- b. Compute the principal curvatures and principal directions for a cylinder with the z axis as its axis and circular cross section of unit radius whose parametrization is given as $\mathbf{r}(u, v) = (\cos v, \sin v, u)$ (10 marks)