MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (COMPUTER AND MATHEMATICS)

BACHELOR OF SCIENCE (COMPUTER AND STATISTICS)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 430: NUMERICAL ANALYSIS II

DATE: 31/7/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Solve the system of equations below using the inverse of the coefficients matrix method (5 marks)

$$x_1 + 2x_2 - x_3 = 2$$
$$3x_1 + 6x_2 + x_3 = 1$$
$$3x_1 + 3x_2 + 2x_3 = 3$$

b) Determine the Eigen values and the corresponding Eigen vectors of the following system. (5 marks)

$$10x_1 + 2x_2 + x_3 = \lambda x_1$$
$$2x_1 + 10x_2 + x_3 = \lambda x_2$$
$$2x_1 + x_2 + 10x_3 = \lambda x_3$$

c) Solve the system of equations below using Doolittle's method.

(5 marks)

$$\begin{aligned} 2x + 3y + z &= 9 \\ x + 2y + 3z &= 6 \\ 3x + y + 2z &= 8 \end{aligned}$$

d) Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss- Elimination method with partial pivoting (5 marks)

e) Consider the following matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

i) Show that the matrix satisfies its own characteristics equation (5 marks)

ii) Determine A^{-1} (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the differential equation $\frac{dy}{dx} = 2e^x y$, $y(0) = 2$. Calculate $y(0.2)$ using Adams predictor corrector formula by calculating $y(0.1)$, $y(0.2)$ *and* $y(0.3)$ using the Euler's modified formula.

(20 marks)

QUESTION THREE (20 MARKS)

Consider the approximate value of $I = \int_{-1}^{1} e^{-x^2} \cos x dx$ obtain the value using.

a) Gauss-Legendre integration method for $n = 2$

(10 marks)

b) Radau integration method for $n = 2$

(10 marks)

QUESTION FOUR (20 MARKS)

a) Consider the system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Where a is a real constant

i. For $a = 0.5$ calculate the value of ω which minimizes the spectral radius of the SOR iteration method.

(3 marks)

ii. Determine the values of a, for which the Jacobi and Gauss-Seidel methods converge.

(7 marks)

b) Calculate the largest Eigen –value for the matrix

$$\begin{bmatrix} 10 & 4 & -1 \\ 4 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

Also evaluate the Eigen vector corresponding to the Largest Eigen -value

(10 marks)

QUESTION FIVE (20 MARKS)

a) Let $f(x) = \ln x$ and $x_0 = 1.8$ for $h > 0$,evaluate $f'(x)$

(5 marks)

b) Calculate $\int_0^{\frac{1}{2}} \frac{x}{\sin x} dx$ using Romberg integration with step size $h = \frac{1}{16}$ (5 marks)

c) Consider the initial value problem (I .v. p) given by

$y' = 2x + 3y \quad ; \; y(0) = 1$

Use Taylors series second order method to get $y(0.4)$ with step size length $h = 0.1$

(10 marks)