MACHAKOS UNIVERSITY

University Examinations 2023/2024 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES
FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE IN APPLIED PHYSICS AND TECHNOLOGY

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SPH 103: MATHEMATICS FOR PHYSICS

DATE: **TIME:**

INSTRUCTIONS:

- The paper consists of **TWO** sections.
- Section **A** is **COMPULSORY** (30 marks).
- Answer any **TWO** questions from section **B** (each 20 marks).

Table 1

Common Derivatives

$\frac{d}{dx}(x)=1$	$\frac{d}{dx}(\csc x)=-\csc x\cot x$	$\frac{d}{dx}(a^x)=a^x\ln(a)$
$\frac{d}{dx}(\sin x)=\cos x$	$\frac{d}{dx}(\cot x)=-\csc^2 x$	$\frac{d}{dx}(\mathbf{e}^x)=\mathbf{e}^x$
$\frac{d}{dx}(\cos x)=-\sin x$	$\frac{d}{dx}(\sin^{-1} x)=\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\ln(x))=\frac{1}{x},\ x>0$
$\frac{d}{dx}(\tan x)=\sec^2 x$	$\frac{d}{dx}(\cos^{-1} x)=-\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\ln\|x\|)=\frac{1}{x},\ x\neq 0$
$\frac{d}{dx}(\sec x)=\sec x\tan x$	$\frac{d}{dx}(\tan^{-1} x)=\frac{1}{1+x^2}$	$\frac{d}{dx}(\log_a(x))=\frac{1}{x\ln a},\ x>0$

SECTION A

QUESTION ONE (30 MARKS)

a) Convert $(\mathbf{1001001100})_2$ to the decimal number equivalent. [3 marks]

b) Solve for x, given that $e^{x+2} = 5e^{2x}$ [3 marks]

c) Express in polar form (r, θ), the complex number $z = \sqrt{3} + i$. [3 marks]

d) Determine the exponential function $f(x) = a^x$ that passes through the points $(3, 8)$ [3 marks]

e) Find the **inverse** of the function $f(x) = ln(x - 5)$ [3marks]

f) Determine the unit vector parallel to the sum of vectors $a = 2i + 4j - 5k$ and $b = i + 2j - 3k$. [3 marks]

g) Find the range and the domain of the function $y = \sqrt{x^2 - 4}$ (***sketch a graph representing the domain and range***) [3 marks]

h) A bag contains 4 red and 3 blue balls. Two draws of 2 balls each are made. Calculate the chance that the first gives 2 red balls and the second draw gives 2 balls if the balls are not returned. [3 marks]

i) Compute the third derivative y''' of the function $y = 5x^3 + 3x^2$. [3 marks]

j) Integrate $\int lnx dx$. [3 marks]

SECTION B

QUESTION TWO (20 MARKS)

a) Solve for **x** given that $log_2 y + log_2 (3y + 4) = 2log_2(3y - 4)$. [4 marks]

b) Solve for **x** in the equation $6^{3x} = 2^{2x-3}$. [3 marks]

c) Find the derivative of the function using chain rule. [4 marks]

$$\frac{d}{dx}\left[(x^2 + 10)^3 + 6\right]^{10}.$$

d) Obtain the first derivative y' of the function. [4 marks]

$$y = x^2 e^{2x^2}.$$

e) Find the slope and an equation of the normal line to the graph of $f(x) = 2x + 3x^2 + 2$ at a point **(1,7).** [5 marks]

QUESTION THREE (20 MARKS)

a) Express $\sin 3x$ in terms of $\sin x$. [6 marks]

b) Differentiate the function $y = x^3 \sin^2 3x$ with respect to **x** [5 marks]

c) Convert the decimal fraction **4.47** to binary number to **5** decimal places. [4 marks]

d) Given that $z_1 = 2 - i$, $z_2 = 1 + 2i$ and $z_3 = 1 - 2i$. Calculate the magnitude of $\left|\frac{z_2 z_1}{z_3}\right|$. [5 marks]

QUESTION FOUR (20 MARKS)

a) The Celsius and Fahrenheit temperature scales are related by $C = \frac{5}{9}(F - 32)$. Where C reading is in degrees and F reading is in Fahrenheit solve for F. and hence solve for F if C is 25 ºC. [3 marks]

b) Evaluate the integral $\int \cos 7x \sin 3x dx$ [5 marks]

c) Evaluate the improper fraction integral $\int \frac{3x^2 - 7x}{3x + 2} dx$ [6 marks]

d) Find the partial differentials $\frac{\partial R}{\partial x}$ and $\frac{\partial R}{\partial y}$ for the function $R_{(x,y)} = \frac{x^2}{y^2 + 1} - \frac{y^2}{x^2 + y}$ [6 marks]

QUESTION FIVE (20 MARKS)

a) Integrate the function using substitution method [6 marks]

$$\int_{-1}^{1} \left(\sqrt{1 + x^3}\right) 3x^2 dx$$

b) Evaluate using integration by parts $\int x \cos x \, dx$ [4 marks]

c) Calculate the triple product $\left|(\vec{A} \times \vec{B}) \times \vec{C}\right|$, given that $\vec{A} = -2\underset{\sim}{i} + 5\underset{\sim}{k}$, $\vec{B} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\vec{C} = 2\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}$ [6 marks]

d) Determine the probability of throwing a total of 7 points or less with two dices. [4 marks]

Table 2

Common Integrals

$\int k\,dx = k\,x + c$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + c$

$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c$

$\int \ln u\,du = u\ln(u) - u + c$

$\int \mathbf{e}^u\,du = \mathbf{e}^u + c$

$\int \cos u\,du = \sin u + c$

$\int \sin u\,du = -\cos u + c$

$\int \sec^2 u\,du = \tan u + c$

$\int \sec u \tan u\,du = \sec u + c$

$\int \csc u \cot u\,du = -\csc u + c$

$\int \csc^2 u\,du = -\cot u + c$

$\int \tan u\,du = \ln|\sec u| + c$

$\int \sec u\,du = \ln|\sec u + \tan u| + c$

$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c$

$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + c$

Table 3

https://trigidentities.info TRIGONOMETRIC IDENTITIES

Co-function Identities

$\sin\theta = \cos(\pi/2-\theta)$
$\sec\theta = \csc(\pi/2-\theta)$
$\tan\theta = \cot(\pi/2-\theta)$

Supplement Angle Identities

$\sin(\pi-\theta) = \sin\theta$ $\quad$ $\csc(\pi-\theta) = \csc\theta$
$\cos(\pi-\theta) = -\cos\theta$ $\quad$ $\sec(\pi-\theta) = -\sec\theta$
$\tan(\pi-\theta) = -\tan\theta$ $\quad$ $\cot(\pi-\theta) = -\cot\theta$

$\sin(\pi+\theta) = -\sin\theta$ $\quad$ $\csc(\pi+\theta) = -\csc\theta$
$\cos(\pi+\theta) = -\cos\theta$ $\quad$ $\sec(\pi+\theta) = -\sec\theta$
$\tan(\pi+\theta) = \tan\theta$ $\quad$ $\cot(\pi+\theta) = \cot\theta$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\csc(-\theta) = -\csc\theta$
$\cos(-\theta) = \cos\theta$ $\quad$ $\sec(-\theta) = \sec\theta$
$\tan(-\theta) = -\tan\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Addition and Subtraction Identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$
$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

Double-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

https://trigidentities.info

Product Identities

$$\sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$
$$\cos A\sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$$
$$\cos A\cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$
$$\sin A\sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

Sum Identities

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$