MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 336: ORDINARY DIFFERENTIAL EQUATIONS II

DATE: **TIME:**

INSTRUCTIONS: Answer **question one** and **any other two** questions

QUESTION ONE (COMPULSORY) (30 MARKS

a) Solve the system of differential equations below using elimination method. (5 marks)

$$\frac{dy}{dt} - 2y + 3x = 0$$
$$\frac{dx}{dt} + 2y + 3x = 0$$

b) Determine the integrability of the differential equation (5 marks)
$zdx + zdy + 2(x + y + \sin z)dz = 0$

c) Consider the differential equation $x^2(1 - x)y'' + (1 - x)y' + y = 0$
   i) Determine the singular points (2 marks)
   ii) Determine the nature of the singular points in (i) above (3 marks)

d) Consider the initial – value problem $\frac{dy}{dx} = 2x^2 + y^2, y(1) = 3$. Determine whether the differential equation has a unique solution (5 marks)

e) Prove that the Bessel function of order zero is given by
$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2.4^2} - \frac{x^6}{2^2.4^2.6^2} + \cdots$ (5 marks)

f) Use Rodrigues formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$ to find $P_2(x)$ (5 marks)

QUESTION TWO (20 MARKS)

a) State the conditions necessary for the existence of the solution of the equation

$\frac{dy}{dx} = f(x, y)$ (6 marks)

b) Obtain a power series solution of the equation $\frac{dy}{dx} = y$ in ascending powers of x (10 marks)

c) Solve the equation $\frac{dy}{dx} = y$ using the method of separation of variables (4 marks)

QUESTION THREE (20 MARKS)

a) Find a linearly independent solution of $(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ by reducing the order (10 marks)

b) Solve $yxdx - z^2dy - xydz$ by treating one of the variables as a constant (10 marks)

QUESTION FOUR (20 MARKS)

a) Locate and classify the singular points of the Legendre differential equation $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$ (5 marks)

b) Show that the solution of a Legendre differential equation $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$ is given by $y = c_0\left[1 - \frac{p(p+1)}{2!}x^2 + \frac{p(p-2)(p+1)(p+3)}{4!}x^4 \ldots\right] + c_1\left[x - \frac{(p-1)(p+2)}{3!}x^3 + \frac{(p-1)(p-3)(p+2)(p+4)}{5!}x^5 \ldots\right]$ (10 marks)

c) Prove that the Legendre polynomial of order 3 is given by $P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Consider the differential equation

$$\frac{d^4y}{dx^4} - 3\frac{d^3y}{dx^3} + 2\frac{dy}{dx} - 8y = 0$$

Reduce the equation to a system of first order differential equation (6 marks)

b) Consider the system below

$$\frac{dx_1}{dt} = x_1 - x_2 - x_3$$
$$\frac{dx_2}{dt} = -x_2 + 3x_3$$
$$\frac{dx_3}{dt} = 3x_2 + x_3$$

i) Express the system in matrix form. (1 mark)

ii) Find the general solutions to the system of differential equations. (10 marks)

iii) Find the particular solution given the initial value $x_1(0) = 0$, $x_2(0) = 0$ and $x_3(0) = 5$ (3 marks)