



**MACHAKOS UNIVERSITY**

**ISO 9001:2008 Certified** 

**SCHOOL OF ENGINEERING AND INFORMATION TECHNOLOGY**

**DEPARTMENT OF CIVIL, ELECTRICAL AND MECHANICAL ENGINEERING**

**UNIVERSITY SUPPLEMENTARY EXAMINATION**

**ECU 104: ENGINEERING MATHEMATICS 1**

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**INSTRUCTIONS:**

- 1) Answer Question ONE and any other TWO questions.
- 2) Mobile phones and any written material are prohibited in the examination room.
- 3) No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.
- 4) All answer booklets should be handed in at the end of the exam whether used or not.
- 5) Programmable calculators are prohibited

### **QUESTION ONE (30 MARKS)**

- (a) The angle between two lines is  $45^\circ$ . If the slope  $m_1$  of  $l_1$  is  $\frac{2}{3}$ . Determine the slope of  $l_2$ . (3mks)
- (b) Find the equation of the circle which has the points  $(0, -1)$  and  $(2, 3)$  as ends of a diameter. (5mks)
- (c) Find the equation of the tangent at the point of contact  $(8, -3)$  of the circle whose equation is  $x^2 + y^2 - 6x + 4y - 12 = 0$ . (5mks)
- (d) Prove that the circles  $x^2 + y^2 - 16x - 12y + 75 = 0$  and  $5x^2 + 5y^2 - 32x - 24y + 75 = 0$  touch each other. (5mks)
- (e) Determine the length of the tangent drawn from the point  $(5, 7)$  to the circle  $x^2 + y^2 - 4x - 6y + 9 = 0$ . (5mks)
- (f) Derive the equation of the parabola with its vertex at  $(3, 2)$  and its focus at  $(5, 2)$ . (4mks)
- (g) Convert the polar equation  $r = 4 \sin \theta$  to rectangular form. (3mks)

### **QUESTION TWO (20 MARKS)**

- (a) Show that the point  $(1, 1)$  is equidistant from the lines  $3x + 4y = 12$ ,  $5x - 12y + 20 = 0$  and  $4x - 3y = 6$ . (6mks)
- (b) Find the equations of the tangent to the curve  $xy = 6$  which are parallel to the curve  $3y + 2x = 0$  (6mks)
- (c) A circle passes through the points  $A(-5, 2)$ ,  $B(-3, -4)$  and  $C(1, 8)$ . Find the point of intersection of the perpendicular bisectors of AB and BC. What is the equation of the circle? (8mks)

### **QUESTION THREE (20 MARKS)**

- (a) Find the equations to the tangents and normal to the parabola  $y^2 = 16x$  at points  $(16, 16)$  and  $(1, -4)$ . The tangents intersect at the point T and normal intersect at R. Prove that the line TR is parallel to the axis of the parabola. (10mks)
- (b) Find the vertex, focus, equation of the directrix and the length of latus rectum if the equation of the parabola is  $3x^2 - 9x - 5y - 2 = 0$ . Hence sketch the curve. (10mks)

### **QUESTION FOUR (20 MARKS)**

- (a) (i) For the vectors  $a = i + 4j - 2k$  and  $b = 2i - j + 3k$  find  $a \times b$ . (3mks)

(ii) Determine the angle between the vectors  $a$  and  $b$  when  $a = i + 2j - 3k$  and

$$b = 2i - j + 4k. \quad (4\text{mks})$$

(b) A force  $F = 10i + 2j - k$  newtons displaces an object from  $A = i + j + k$  to  $B = 2i - j + 3k$  (in metres). Find the work done. (5mks)

(c) Find the moment and magnitude of the moment of force of  $(i + 2j - 3k)$  newtons about point  $b$  having coordinates  $(0,1,1)$  when the forces acts on a line through  $A$  whose coordinates are  $(1,3,4)$ . (8mks)

### **QUESTION FIVE (20 MARKS)**

(a) Find the coordinates of the centre, vertices and foci, the equations of the diretrices, the equations of the asymptotes and sketch the graph of the hyperbola whose equation is  $9x^2 - 16y^2 - 18x - 64y - 199 = 0$ . (10mks)

(b) Find the equations of the tangent and normal to the ellipse  $5x^2 + 3y^2 = 137$ , at the point in the first quadrant whose  $y$  coordinate is 2. (6mks)

(c) Derive the equation of the ellipse having its centre at the origin, one focus at  $(0,3)$  and the length of semi-major axis 5. (4mks)