MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR BACHELOR OF EDUCATION

SMA 402: FIELD THEORY

DATE: 23/7/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS: Answer questions ONE and any other TWO questions.

NSTRUCTIONS TO CANDIDATES

(a) Answer ALL the questions in Section A and ANY TWO Questions in Section B

Question one (30 marks)

a) Define the following.

i) Ring

ii) Field

iii) Characteristic of a ring

iv) An integral domain (5 marks)

b) Show that f(x) = $x^2 - 3$ is irreducible over Q (3 marks)

c) Prove that if K| F be a field extension. If this extension is finite then it is an algebraic extension (6 marks)

d) Draw the subfield lattice for a field of (4 marks)

i. 125 elements

ii. 7^{12} elements

e) Show that suppose E is an extension of degree n over a finite field F. If F has Q elements then E has a q^n elements. (4 marks)

f) Prove that, if $f(x) \in F(x)$ and $F(x)$ can be of degree 2 or 3, then $f(x)$ is reducible if it has a zero in F (3 marks)

g) Find the minimal polynomial of

i. $2+\sqrt{5}$ over $\mathbb{Q}$
ii. $e^{2\pi i/3}$ over R
iii. $e^{2\pi i/3}$ over C

QUESTION TWO (20 marks)

a) Find the group of G(F|Q) if $F=Q(\sqrt{2},\sqrt{3})$ (4 marks)

b) Given that $f(X) = (x^2-3)(x^3+1)$. Find the splitting field of f(X) over Q (10 marks)

c) Show that the polynomial $x^2 - 2x - 1$ and $x^2 - 2$ has the same splitting field over Q (6 marks)

QUESTION THREE (20 MARKS)

a) Show that $f(x) = x^4 - 4x^2 - 5$ splits over $k = Q(i, \sqrt{5})$ (3 marks)

b) Show that $x^3 + x + 1$ is irreducible in $\mathbb{Z}_2[x]$. Write the multiplication and addition table for it (6 marks)

c) Determine if the following functions are reducible or irreducible over the given fields

i) $f(x) = x^2+x+1$ in$\mathbb{Z}_2[x]$ (2 marks)
ii) $f(x) = x^2+x+1$ in $\mathbb{Z}_3[x]$ (2 marks)
iii) $f(x) = x4 + x + 1$ in $\mathbb{Z}_2[x]$ (2 marks)

d) Find the basis of $Q(\sqrt{2}, \sqrt{3})$ over Q (5 marks)

QUESTION FOUR 20 MARKS)

a) Show that $G(Q(\sqrt[3]{2})|Q$ is identity. Hence show that the only automophism of $Q(\sqrt[3]{2})$ over Q is the identity. (10 marks)

b) State and prove the Eisenstein's irreducibility criterion (10 marks)

QUESTION FIVE (20 MARKS)

a) Let $f(x) = a_0 + a_1x + \cdots + a_0x^n \in \mathbb{Z}(x)$be of degree n and $a_n \neq 0$. Let $u/v \in Q$be a root of f(x), where u and v are relatively prime. Then $u/a_0$and v/a_n. prove (10mks)

b) Show that the cyclotonic polynomial $Qp(x) = 1 + x + \cdots + x^{p-1} = \frac{(x^{p-1})}{(x-1\,)}$ irreducible in $\mathbb{Z}(x)$ where p is prime. (10mks)