MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

.... SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF ....

SMA 303: RING THEORY

DATE: **TIME:**

INSTRUCTIONS:

Answer QUESTION ONE and Any other TWO Questions

QUESTION ONE

(a) (i) Define what is meant by a zero divisor in a ring. (2 marks)

(ii) Find all the Zero divisors in Z12 (3marks)

(b) (i) Define what is meant by a unit in a ring (2 marks)

(ii) Find all the units in Z8 (3 marks)

c) (i) Define what is meant by an ideal I of a Ring R (2 marks)

(ii) Find all the ideals in Z6. (3 marks)

d) (i) Find all the cosets in the quotient ring 3Z/12Z. (2 marks)

(ii) Show that if R is a ring with unity and I is an ideal of R containing a unit, Then I=R. (3 marks)

e) (i) Define what is meant by a subset S Is a subring of a ring R (2 marks)

(ii) Give an example of a subring of a ring which is not an ideal (3 marks)

f) (i) Let R and S be two rings and $f:R\rightarrow S$ be a mapping. Define what is meant by f is a ring homomorphism. (2 marks)

(ii) Let I be an ideal of a ring R. Show that the mapping $f:R\rightarrow R/I$ defined by $F(a)=a+I$ for all a€ R is a ring homomorphism (3 marks)

QUESTION TWO

a) If Z_m is a ring of integers modulo m, then the set M2 (Z_m) of all 2x2 matrices over Z_m is a ring (you need not to verify this).

(i) How many elements are there in $M_2(Z_m)$? (2 marks)

ii) Show the multiplication is not commutative in $M_2(Z_2)$ (2 marks)

(iii) Which elements of M2(Z_2) have a multiplicative inverse? (3 marks)

b) Let $f:R\rightarrow S$ be a ring homomorphism from a ring R into a ring S.

(i) Define what is meant by the kernel of £. (2 marks)

(ii) Show that ker f is an ideal of R (2 marks)

(iii) Show that f is one-to one if and only if ker f={0} (3 marks)

c) (i) Show that every field is an integral domain (4 marks)

(ii) Give an example of an integral domain which is not a field (2 marks)

QUESTION THREE

a) (i) Define what is meant by a maximal ideal (2 marks)

(ii) Define what is meant by a prime ideal (2 marks)

b) (i) List all the maximal ideals of Z_{12} (3 marks)

(ii) List all the prime deals of Z (3 marks)

c) (i) Let R be a commutative ring with unity, and let $N \neq R$ be an ideal in R.

Show that R/N is an integral domain if and only if N is a prime ideal in R.

(4 marks)

(ii) Show that every maximal ideal in a commutative ring R with unity is a prime ideal. (2 marks)

(iii) Show that Z is a principal ideal domain. (4 marks)

QUESTION FOUR

$S=\{(a, b)|a, b € Q\}$, with operations + and . forms a ring where the operations + and . are defined by $(a, b)+(c, d)=(a+c, b+d)$

$(a,b).(c,d)=(ac+bc+ad,bd)$

(a) Show that S is a commutative ring with unity e. (6 marks)

(b) Find all the elements in S of the form x.x=e (6 marks)

(c) Determine the multiplicative inverse for (a,b) in S when it exists and the Relationship between a and b when the inverse does not exist. (6 marks)

(d) Is S a field? Give reasons. (2 marks)

QUESTION FIVE

(a) Show that if F is a field, then every ideal in F[x] is principal (5 marks)

(b) Compute the sum and product of the polynomials X^3+3x^2+2x+1 and X^2+4x+2 in $Z_5[x]$. (5 marks)

(c) Find the quotient and the remainder when $x^5+x^4+2x^3+x^2+4x+2$ is divided by X^2+2x+3. (5 marks)

(d) Find the GCD of x^3+x^2+x+1 and x^2+2 in $Z_3[x]$ and express the result in the form

$\gamma(x)(x^3+x^2+x+1)+\mu(x)(x^2+2)$

Where $\gamma(x)$ and $\mu(x)$ are polynomials in $Z_3[x]$. (5 marks)