MACHAKOS UNIVERSITY

University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (ECONOMICS AND STATISTICS)

BACHELOR OF EDUCATION (SPECIAL EDUCATION)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF SCIENCE (STATISTICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR (ARTS)

SMA 102: BASIC MATHS

DATE: 31/1/2022 **TIME: 8:30 – 10:30 AM**

Instructions to Candidates.

Answer question ONE and any other TWO questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Express with real denominator the value of: $z = \frac{1}{5+4i} + \frac{1}{5-4i}$. (3 marks)

b) Determine whether the following is a tautology, a contradiction or neither by constructing a truth table. $(Q \Rightarrow P) \wedge (\sim Q \vee P)$ (3 marks)

c) Factorize $2x^3 - x^2 - 2x + 1$. Hence find the values of x for which $2x^3 - x^2 - 2x + 1 > 0$. (4 marks)

d) The polynomial $f(x) = 2x^3 + Ax^2 + Bx - 3$ is exactly divisible by $(x - 1)$ and has a remainder $+9$ when divided by $(x + 2)$. Find the values of the constants A and B. Hence solve the equation $f(x) = 0$. (7 marks)

e) All the students in an academy of 30 musicians play either the piano or the violin. In this academy 22 play the piano and 17 play the violin. Display this information in a Venn diagram hence find how many students play either the piano or violin but not both. (5 marks)

f) The coefficient of x in the expansion of $(1+ax)^5$ is equal to the coefficient of x^4 in the expansion of $\left(9+\frac{x}{3}\right)^6$, calculate the value of a. (5 marks)

g) Use the method of induction to show that: $1+3+5+\ldots+(2n-1)=n^2$. (3 marks)

QUESTION TWO (20 MARKS)

a) How many arrangements of the letters in the word $PERMUTING$ start with a vowel? (3 marks)

b) When divided by $(x-1)$ the polynomial $g(x)=ax^3-3x^2+bx+6$ leaves a remainder of -6 and when divided by $(x+2)$ it leaves a remainder of zero. Find the values of the constants a and b. Hence solve the equation $ax^3-3x^2+bx+6=0$. (7 marks)

c) In the binomial expansion of $\left(1+\frac{x}{n}\right)^n$ in ascending powers of x, the coefficient of x^2 is $\frac{7}{16}$. Given that n is a positive integer,

i. Find the value of n.

ii. Evaluate the coefficient of x^3 in the expansion. (5 marks)

d) Use induction to check whether the following mathematical conjecture holds for all of $n \in \mathbb{N}$

$\frac{1}{1\times2}+\frac{1}{2\times3}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$. (5 marks)

QUESTION THREE (20 MARKS)

a) Given that $A=\{2\}$, $B=\{x,y\}$,$C=\{a,b,c\}$. Determine the product set $A\times B\times C$. (3 marks)

b) For three intersecting sets W, X, and Y, draw a Venn diagram and shade the region corresponding to:

i. $W\cap(X\cup Y)$. (3 marks)

ii. $W-(X-Y)$ (3 marks)

c) If $M$and N are sets use element argument method or Formal proof to show that $M-N=M\cap N^c$. (3 marks)

d) A survey was conducted on the first 41 presidents of the United States of America. The following information was gathered:

8 held cabinet posts,

14 served as vice-president,

15 served in the U.S.A Senate,

2 served in cabinet posts and as vice-president,

4 served in cabinet posts and in the U.S. Senate,

6 served in the U.S. Senate and as vice-president,

1 served in all three positions.

Required: Use a Venn diagram to determine how many presidents served in:

i. None of these three positions? (2 marks)

ii. Only in the U.S.A Senate. (2 marks)

iii. At least one of the three positions. (2 marks)

iv. Exactly two positions. (2 marks)

QUESTION FOUR (20 MARKS)

a) State De Moivre's Theorem. (1 mark)

b) In polar form determine the roots of the equation $z^2 - 12z + 52 = 0$. Hence represent the roots z_1 and z_2 in an Argand diagram. (8 marks)

c) Find the square roots of $-7 + 24i$ by equating the real and imaginary parts of $-7 + 24i = (x + yi)^2$. (5 marks)

d) Find the principal value of the argument of the complex number $z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$. Hence write z in polar form, then use this expression to determine $z^{\frac{1}{3}}$ and z^4 . (6 marks)

QUESTION FIVE (20 MARKS)

a) In a certain country vehicle license plates consist of two letters of the alphabet followed by four digits from 0 to 9 inclusive.

i. How many different license plates are there if the second letter on the plate is either $''A''$ or $''K''$and the last digit is either a$''2''$ or a$''9''$? (4 marks)

ii. How many different license plates are there if no restrictions are imposed? (4 marks)

b) A group of tourists is composed of six from Tokyo, seven from Moscow and eight from Nairobi.

i) In how many ways can a committee of six tourists be formed with two tourists from each city? (4 marks)

ii) In how many ways can a committee of seven tourists be formed with at least two tourists from each city? (4 marks)

c) Simplify completely $\left(cos\frac{7}{3}\pi + i\,sin\frac{7}{3}\pi\right)\left(cos\frac{2}{3}\pi - i\,sin\frac{2}{3}\pi\right)$ leaving your answer in cartesian/rectangular form. (4 marks)