MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS & COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 302: GROUP THEORY

DATE: 17/8/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Define the following terms
   i) Binary operation
   ii) Transposition
   iii) Permutation
   iv) Set
   v) Function (5 marks)

b) Construct the multiplication table for A_3. (5 marks)

c) Construct the addition table for Z_{12} and show that $\{0,5\}$ $and\{0,1\}$ are not the proper subgroups and construct its lattice diagram. (6 marks)

d) Every cyclic group is abelian. Proof (5 marks)

e) If G is a group with binary operation * and if a and b are any elements of G then the linear equation $y * a = b$ and $x * a = b$ have a unique solution in G. proof. (5 marks)

f) If $f: G \to G'$ is an isomorphism of G with G' and e is the identity of G then f(e) is the Identity in G'. (4 marks)

QUESTION TWO (20 MARKS)

a) Define a group (5 marks)

b) Let A be a non-empty set and let S_A be the collection permutations of A. Then S_A is a group under permutation multiplication. Proof. (7 marks)

c) Construct addition and the multiplication table Z_{16} ,find its subgroups and draw it lattice diagram. (8 marks)

QUESTION THREE 20 MARKS

a) Construct the multiplication table for D_4 (8 marks)

b) The collection of all even permutations of a finite set of n elements form a subgroup of order $\frac{n!}{2}$ proof. (6 marks)

c) Let G be a group and let $a \in G$. The $H = \{a^n | n \in Z\}$ is a subgroup of G and is the smallest subgroup which contains a. proof (6 marks)

QUESTION FOUR 20 MARKS

a) If G is a group with binary operation * then the right and the left cancellation hold in G. proof . (6 marks)

b) Define a function $f: G \to G^1$ by $f(x) = axa^{-1}$ show that f is isomorphism. (6 marks)

c) Construct the multiplication table for Klein 4-group and is $\{e, a, b\}$, $\{e, a, c\}$ and $\{e, b, c\}$ a subgroup of klein4-group. Construct the lattice diagram. (8 marks)

QUESTION FIVE 20 MARKS

a) Let G be a group and H a non empty subset of G. Then H is a subgroup of G if and only if for $, b \in H, ab^{-1} \in H$. prove (6 marks)

b) In a group G with the operation * , there is only one identity and inverse (6 marks)

c) State whether the following permutation is odd or even

i) $(13245)(679)(1011)$ (4 marks)

ii) $(134)(587)(26)$ (4 marks)