



MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN

BUILDING, CIVIL AND ELECTRICAL AND ELECTRONICS ENGINEERING

SMA 404: COMPLEX ANALYSIS II

DATE:

TIME: 2 Hours

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION 1: COMPULSORY (30 MARKS)

a) Evaluate $\lim_{z \rightarrow 2} \frac{z^2 - 2z + 4}{z - 2}$ (3 marks)

b) Calculate the residues for $f(z) = \frac{1}{z(z+1)}$. (4 marks)

c) Given that $w = f(x) = (z^2 + 1)^{\frac{1}{2}}$. Show that $z = \pm i$ are branch points of $f(x)$ (6 marks)

d) Prove that the function $f(x) = z^2$ is analytic (7 marks)

e) Find the image of the line $y = \frac{1}{2}x + 2$ (10 marks)

QUESTION 2 (20 MARKS)

a) Explain the nature of the transformation $w = z^2$ (5 marks)

b) Find the image of the point (4,3) on z -plane under the transformation $w = 2z^2 + 3$ (5 marks)

- c) Investigate for convergence or divergence of the series $\sum_{n=0}^{\infty} \frac{(z+3-i)^n}{4^n (n+1)^3}$ (10 marks)

QUESTION 3 (20 MARKS)

- a) Find the Laurent expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ for $1 < |z| < 2$ (6 marks)

- b) Solve $z^4 = i$ (7 marks)

- c) Show that $f_1(z) = \sum_{n=0}^{\infty} \left(\frac{z-i}{1-i} \right)^n$ and $f_2(z) = \sum_{n=0}^{\infty} z^n$ are analytic continuation of each other.

Indicate the region on the Argand diagram (7 marks)

QUESTION 4 (20 MARKS)

- a) Find the fixed points of the transformation $T(z) = z$ (6 marks)

- b) Find the Mobius transformation which maps the points $z_1 = 2$, $z_2 = i$ and $z_3 = -2$ onto the points $w_1 = 1$, $w_2 = i$ and $w_3 = -1$ respectively. (7 marks)

- c) Consider the lines $l_1 : x = 2$ and $l_2 : y = 2$ meeting at $z = z_0 = (2, 2)$. Discuss conformality of $T(z = z^2)$ at z_0 . (7 marks)

QUESTION 5 (20 MARKS)

- Show that $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2 + 1)^2} dx = \frac{2\pi}{e^3}$. (20 marks)