



MACHAKOS UNIVERSITY

ISO 9001:2008 Certified



**SCHOOL OF PURE AND APPLIED SCIENCES FOR THE BACHELOR'S
DEGREE**

DEPARTMENT OF ELECTRICAL ENGINEERING

EEE 402: COMPLEX ANALYSIS FOR ENGINEERING

INSTRUCTIONS:

- 1) **Answer Question ONE and any other TWO questions.**
- 2) **Mobile phones and any written material are prohibited in the examination room.**
- 3) **No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.**
- 4) **All answer booklets should be handed in at the end of the exam whether used or not.**
- 5) **Programmable calculators are prohibited**

QUESTION ONE (COMPULSORY) (30 MARKS)

- a). Define the following terms
- i. A complex number (1 mark)
 - ii. Analytic function (1 marks)
- b). Test the continuity of a complex function
- $$f(z) = \begin{cases} \frac{z^2 + 4}{z - 2i} & , z \neq 2i \\ 3 + 4i & , z = 2i \end{cases} \quad (4 \text{ marks})$$
- c). Determine the singular points of the following function and residues at each point.
- $$f(z) = \frac{z^2}{(z-1)^2(z+2)}$$
- and hence evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where $C : |z| = 3$. (6 marks)
- d). Find all values of $(i)^{1+i}$ hence state the principle value (3 marks)
- e). Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ $C : |z+1| = 3$, using Cauchy's theorem (5 marks)
- f). Let $u(x, y) = x^2 - y^2 - 2xy - 2x - y - 1$. Find $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is harmonic. Express $f(z)$ in terms of z . (6 marks)
- g). Evaluate $\int_0^{1+i} (x - y + ix^2) dz$ along the straight line from $z = 0$ to $z = 1 + i$ (4 marks)

QUESTION TWO (20 MARKS)

- a) Find all values of z such that $e^z = 1 + i\sqrt{3}$ (6 marks)
- b) Show that $\cos^{-1}(z) = -i \ln(z + i\sqrt{1-z^2})$. Hence find all solutions to the equation $\cos z = \sqrt{2}$ (7 marks)
- b) Show that $|\cosh z|^2 = \cos^2 y + \sinh^2 x$ (7 marks)

QUESTION THREE (20 MARKS)

- a) Derive the Cauchy-Riemann conditions for analytic function. (7 marks)
- b) Show that $f(z) = \ln z$ is analytic function (6 marks)

- c) Show that $v(x, y) = \ln(x^2 + y^2) + x - 2y$ is harmonic. Find $u(x, y)$ such that $u(x, y) + iv(x, y)$ is analytic. Express $f(z)$ in terms of z . (7 marks)

QUESTION FOUR (20 MARKS)

- a) Let $f(z)$ be analytic in the simply connected region R , let C be a simple closed contour that lies in R . If $z = a$ is a point that lies interior to C , show that

$$f'''(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)^3} dz. \quad (7 \text{ marks})$$

- b) Evaluate the following integral $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ using Cauchy's integral formula where $C: |z-i| = 2$. (7 marks)

- c) Use De-Moivre's theorem to show that $\cos 4\theta = 8\sin^4 \theta - 8\sin^2 \theta + 1$ (6 marks)

QUESTION FIVE (20 MARKS)

- a) Find the fifth roots of the complex number $-4 + 4i$. (7 marks)

- b) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1) dz$ along the curve $x = t + 1$, $y = 2t^2 - 1$ (6 marks)

- c) Evaluate the following integral $\int_C \frac{12z-7}{(z-1)^2(2z+3)} dz$ using Cauchy's residue theorem where $C: |z+i| = \sqrt{3}$. (7 marks)