



MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF ECONOMICS AND STATISTICS

SMA 361: THEORY OF ESTIMATION

DATE: 18/4/2019

TIME: 11:00 – 1:00 PM

INSTRUCTION: Answer Question **ONE** which is compulsory

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Explain the meaning of the following terms as applied in theory of estimation
- (i) Population
 - (ii) An estimator (4 marks)
- b) Let $x_1, x_2, \dots, x_n$ be a random sample drawn from a population with mean μ (unknown). Show that if $y = \sum_{i=1}^n a_i x_i$ is unbiased estimator for μ if $\sum_{i=1}^n a_i = 1$ (6 marks)
- c) Let $x_1, x_2, \dots, x_n$ denote a random sample from a pdf is $P(X = x) = \frac{e^{-\mu} \mu^x}{x!}$, $x=0,1,\dots$
- Use the factorization theorem to show that $T = \sum x_i$ is a sufficient estimator of μ (4 marks)
- d) Let $x_1, x_2, \dots, x_n$ be a random sample from a population with mean μ and variance δ^2 .

Consider the following estimators for μ .

$$t_1 = \frac{1}{2} (x_1 + x_2)$$
$$t_2 = \frac{\frac{1}{2} x_1 + x_2 + \dots + x_{n-1}}{2(n-1)}$$
$$t_3 = \bar{x}$$

- i) Show that each of 3 estimators is unbiased. (5 marks)
- ii) Determine the efficiency of t_3 relative to t_2 (5 marks)
- e) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a normally distributed population with mean μ (unknown) and known variance σ^2 . Determine the maximum likelihood estimator (MLE) of μ . (6 marks)

QUESTION TWO (20 MARKS)

- a) Use the following data to fit the regression line $y = \alpha + \beta x$

x	-2	-1	0	1	2
y	0	0	1	1	3

(8marks)

- b) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a population whose pdf is given by

$$f(x, \theta) = \theta e^{-\theta x}, x > 0,$$

Determine the Cramer-Rao lower bound for θ . (12marks)

QUESTION THREE (20 MARKS)

- a) Let $x_1, x_2, \dots, x_n$ be a random sample from $x \sim N(\mu_0, \sigma^2)$ where μ, σ^2 are both unknown. Determine a joint sufficient statistic of μ , and σ^2 (8 marks)
- b) A random sample of size n is drawn from a normal population with $(0, \sigma^2)$. Determine the minimum variance unbiased estimator (MVUE) of σ^2 and its variance. (12 marks)

QUESTION FOUR (20 MARKS)

- a) Let $x_1, x_2, \dots, x_n$ denote a random sample from a pdf

$$f(x) = \begin{cases} (\theta + 1)x^\theta, & 0 < x < 1, \theta > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Obtain the moment estimator of θ (8 marks)

- b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased.

(12 marks)

QUESTION FIVE (20 MARKS)

- a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 99% confidence intervals for the true mean. (8 marks)
- b) If $x_1, x_2, \dots, x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x, \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$ (12 marks)