MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEED EDUCATION)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ARTS

SMA 104: CALCULUS I

DATE: 10/8/2021 **TIME: 11.00-1.00 PM**

INSTRUCTIONS:

Answer question one in section A and any other TWO questions in section B.

SECTION A

QUESTION ONE (30 MARKS)

a) Consider the function $f(x) = 3x^2 + 2x - 1$. Show that $f(x)$ is continuous at the point $x = 2$ (4 marks)

b) Evaluate $\lim_{x \to \infty} \left[\frac{3x-2}{\sqrt{2x^2+1}} \right]$ (4 marks)

c) Evaluate the value of $\sqrt{101}$ using the technique of small changes approximation. (4 marks)

d) Given that $y = \ln(\sqrt[3]{x^2 - 3x + 1})$. Determine $\frac{dy}{dx}$ (4 marks)

e) Eliminate the parameter t from the equations $x = \sin t$, $y = \cos 2t$ hence verify that

$$\frac{d^2 y}{dx^2} + 4 = 0$$ (4 marks)

f) Consider the function $f(x) = x^3 + x^2 - 5x$. Using the first derivative test, determine its;

i. Relative maxima point (4 marks)

ii. Relative minima point (2 marks)

g) A petrol tanker is damaged in a road accident and petrol leaks onto a flat section of motorway. The leaking petrol begins to spread in a circle of thickness $2mm$. Petrol is leaking from the tanker at a rate of $0.0084m^3/s$ Find the rate at which the radius of the circle of petrol is increasing at the instant when the radius of the circle is $3m$ giving your answer in m/s to 2 decimal places.

(4 marks)

QUESTION TWO (20 MARKS)

a) Differentiate the function: $f(x) = \ln\left[\frac{x^2}{(x+3)(x^2-2)}\right]$ (5 marks)

b) Find the equation of the tangent and normal at the point $(2,1)$ on the curve: $y^2 + 3xy - 2x^2 + 1$

(6 marks)

c) A vessel containing water is in the form of an inverted cone with semi-vertical angle of 30^0. There is a small hole at the vertex of the cone and the water is running out at a rate of $3cm^3$ per second. Calculate the rate at which the surface area in contact with water is changing when there are $81\pi cm^3$ of water remaining in the cone. (9 marks)

QUESTION THREE (20 MARKS)

Determine;

a) $\lim\limits_{x \to +\infty} \left(2x^{11} - 5x^6 + 3x^2 + 1\right)$ (6 marks)

b) $\lim\limits_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$ (5 marks)

c) $\lim\limits_{x \to 0} \frac{e^{\tan x} - e^x}{\tan x - x}$ (5 marks)

d) $\lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Given two functions: $f(x) = 5x - 3$, $g(x) = 7 - \frac{1}{6}x$ find the following:

i. $g(f(x))$ the composition of g and f (3 marks)

ii. $f^{-1}(x)$ the inverse of $f(x)$ (3 marks)

b) Find derivatives of each of the following:

i) $y = \sin^{-1}\sqrt{x}$ (2 marks)

ii) $f(x) = \ln(\cosh x)$ (2 marks)

c) The height of an object attached to a spring is given by the harmonic equation:

$$y = \frac{1}{3}\cos 12t - \frac{1}{4}\sin 12t$$

where y is measured in inches and t in seconds.

i. Calculate the height and velocity of the object when $t = \frac{\pi}{8}$ seconds. (5 marks)

ii. Show that the maximum displacement of the object is $\frac{5}{12}$ inches. (5 marks)

QUESTION FIVE (20 MARKS)

Consider the function $f(x) = x^5 - 5x^3$

a) using the second derivative test, determine;

i. Local maximum point (3 marks)

ii. Local minimum point (3 marks)

b) Work out the coordinate of the points of inflection of $f(x)$ (4 marks)

c) Discuss the concavity of $f(x)$ (5 marks)

d) Sketch the graph of $f(x)$ (5 marks)