

Machakos University
SCHOOL OF PURE AND APPLIED SCIENCES
 University Supplementary Examinations
BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER
SCIENCE/STATISTICS & PROGRAMMING/MATHEMATICS)
SMA 261: PROBABILITY AND STATISTICS II

DATE:

TIME:

Question One. Compulsory (30 marks)

(a) If the joint probability function for x_1 and x_2 is given by $f(x, y) = \frac{1}{30}(x + y)$

$$x = 0, 1, 2, 3.$$

$$y = 0, 1, 2$$

(i) Show that $f(x, y)$ is a probability distribution (3 marks)

(ii) Hence or otherwise compute $p(x \leq 2)$ (3marks)

(b) Given that $f(x, y) = \begin{cases} \frac{3}{2}y^2, & 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine the marginal probability function for x and y (6marks)

(c) Consider the bivariate table below

		y		
		-1	0	1
x	-1	0.04	0.01	0.20
	1	w	0.03	0.60

(i) Determine the value of w (2marks)

(ii) Determine the marginal density of x (3marks)

(d).The joint pdf of x and y is given by $f(x, y) = bxy$, for $0 < x, y < 1$.

Determine the value of b (4marks)

(e).Show that $E(X+Y)=E(X)+E(Y)$ if X and Y are independent continuous random variables (5marks)

(f). If x and y are independent random variables with a joint p.d.f defined by:

$$f(x, y) = \begin{cases} 4xy & 0 < x, y < 1, \\ 0 & \text{elsewhere} \end{cases}$$

Determine the $E(x, y)$ (4marks)

Question Two

(a) The joint probability density function for x and y is given by $f(x, y) = \frac{1}{54}(x + y)$

$x = 1, 2, 3.$

$y = 1, 2, 3, 4$

- (i) Determine the conditional distribution of y given that $X=x$ (5marks)
- (ii) Hence calculate $p(y = 4/x=3)$ (3 marks)

(b). Suppose that x and y are discrete random variables with a p.d.f as shown in the bivariate table below:

x	y		
	0	1	2
0	0.1	0.1	0.2
1	0	0.15	0.05
3	0.1	0.2	0.1

Determine the covariance of x and y ie $\text{cov}(x, y)$. (12marks)

Question Three

(a) Given x and y have the joint probability density function (pdf) given by

$$f(x, y) = \begin{cases} K(x^2 + y^2), & 0 \leq x \leq 1; 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Determine (i) the value of the constant K . (3 marks)

(ii) the conditional p.d.f of x given for any value of y (5marks)

(iii) $\text{pr}(x < \frac{1}{2} / y = \frac{1}{2})$ (2marks)

(b). Suppose X is a continuous and Y is discrete random variables with a joint p.d.f given by:

$$f(x, y) = \frac{yx^{y-1}}{3}, 0 < x < 1, y \in 1, 2, 3. \text{ Determine}$$

- (i) $f_y(y)$ (5marks)
- (ii) $E(X)$ (5marks)

Question Four

- (a) The joint pdf of x and y is given by $f(x, y) = \begin{cases} 2e^{-(x+2y)}, & x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$

Determine whether or not x and y are independent. (6 marks)

- (b) The joint pdf of x and y is given by that $f(x, y) = \begin{cases} 6x^2y, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

- Determine: (i) μ_x and μ_y (4 marks)
(ii) d_x^2 and d_y^2 (4 marks)
(iii) The correlation coefficient between x and y (6 marks)

Question Five

- (a) Given that x and y are independent random variables with a joint pdf defined by that

$$f(x, y) = \begin{cases} 1, & 0 < x, y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the joint probability density function of $v = x + y$ and $v = x - y$. (10 marks)

- (b) Given that $E(y/x) = \alpha + \beta x$, and $E(X/Y) = \varphi + \delta y$ where E denotes expectation, x and y denotes random variables and $\alpha, \beta, \varphi, \delta$ are constants. Show that:

$$\frac{\alpha}{\varphi} = \frac{1-\beta}{1-\delta} \quad (10 \text{ marks})$$