



MACHAKOS UNIVERSITY

University Supplementary Examinations

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

**SECOND YEAR FIRST SEMESTER EXAMINATION FOR BACHELOR OF
EDUCATION, BACHELOR OF SCIENCE AND BACHELOR OF ARTS**

SMA 202: LINEAR ALGEBRA I

DATE:..... TIME:.....

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION ONE 30 Marks (Compulsory)

a.) Let A and B be $n \times n$ skew symmetric matrices, namely $A^T = -A$ and $B^T = -B$. Prove that;

i.) $A + B$ is skew symmetric (2marks)

ii.) AB is skew symmetric (3marks)

b.) Show that $(A - B)(A + B) = A^2 - B^2$ iff $AB = BA$ (3marks)

c.) Evaluate $\begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 3 \end{vmatrix}$ (4marks)

d.) Given the system

$$x + 2y = 6$$

$$4x + 3y = 3$$

Solve the system using crammers rule (4marks)

e.) Show that the vectors $u = (\sin \theta, \cos \theta)$ and $v = (\cos \theta, -\sin \theta)$ are orthogonal (4marks)

f.) Determine the equation of the plane through the points

$A(1, 1, 1)$, $B(5, 0, 0)$ and $C(3, 2, 1)$. (5marks)

g.) Consider the matrix $A = \begin{pmatrix} -4 & 1 & -6 \\ 1 & 2 & -5 \\ 6 & 3 & -4 \end{pmatrix}$. Reduce the matrix to echelon form. (5marks)

Question two (20marks)

- a. Solve the following system of equations using the Crammer's rule (8marks)
- $$\begin{aligned}x + 2z &= 6 \\ -3x + 4y + 6z &= 30 \\ -x - 2y + 3z &= 8\end{aligned}$$
- b. Determine the inverse of the following matrix if possible. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ (7marks)
- c. Determine the equation of the plane perpendicular to the line $\mathbf{n} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ which passes through the point $A(1,2,1)$ (5marks)

Question Three (20marks)

- a.) Show that the line $y = x + 1$ is not a subspace of $\mathfrak{R}$ i.e
 $W = \{(x, y) / y = x + 1\} = \{(x, x + 1) / x \in \mathfrak{R}\}$ (5marks)
- b.) Calculate the area of a triangle having vertices at
 $P(1, 3, 2)$, $Q(2, -1, 1)$ and $R(-1, 2, 3)$ (5marks)
- c.) Determine the value of h such that $(1, 3, 3)$, $(-2, -6, h)$ and $(3, 9, -1)$ are linearly independent (5marks)
- d.) Calculate the work done in moving an object a long a vector $\vec{A} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ if the applied force is $\vec{F} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$ (5marks)

Question four (20marks)

- a) Solve the system of linear equations below by Gauss-Jordan elimination method. (7marks)
- $$\begin{aligned}2x + 3y - 6z &= 11 \\ x - 2y + 3z &= 9 \\ 3x + y &= 7\end{aligned}$$
- b) Use Gram-Schmidt processes to find an orthogonal basis spanning the same subspace of $\mathbb{R}^n$ as the vectors $\langle 0, 2, 1, -1 \rangle$, $\langle 0, -1, 1, 6 \rangle$ and $\langle 0, 2, 2, 3 \rangle$ (7 marks)
- c) Find the basis and dimension of the set spanned by the vectors $(1, -1, 2)$, $(0, 5, -8)$, $(3, 2, -2)$, and $(8, 2, 0)$. (6 marks)

Question four (20marks)

a.) Solve the following system using Crammers method

$$2x - 5y + 2z = 7$$

$$x + 2y - 4z = 3$$

$$3x - 4y - 6z = 5$$

(10 marks)

b.) Consider the system

$$x_1 + 3x_2 + 2x_3 = 3$$

$$4x_2 + 2x_1 + 2x_3 = 8$$

$$x_1 + 2x_2 - x_3 = 10$$

Solve using matrix inversion method

(10marks)