MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
BACHELOR OF SCIENCE IN COMPUTER SCIENCE
BACHELOR OF SCIENCE IN CLOUD COMPUTING
BACHELOR OF SCIENCE IN MATHEMATICS

SMA110/SCO 108/SST 102: DISCRETE MATHEMATICS

DATE: 21ST JANUARY, 2025 **TIME: 2.00 PM – 4.00 PM**

INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

a) Define the following terms.
   i). Symmetric difference
   ii). Relative complement
   iii). Arithmetic progression
   iv). Spanning subgraph
   v). trail (5 marks)

b) Determine the power set of the set $A = \{a, b, c, d\}$. (3 marks)

c) Construct the logic circuit for $Y = (XY + XZ')' + (X'Y + YZ)$ (5 marks)

d) Construct the truth table for the Conjunction of two proposition (4 marks)

e) In how many distinguishable ways can the product $P^8X^7Y^8Z^7W^6$ be arranged without using exponents. (3 marks)

f) Given $A = \big((abc)'c\big)'\big(a' + c\big)(b' + ac')'$ express it as a sum of product expression. .
(5 marks)

g) Prove that a bipartite graph has no triangles. (5 marks)

QUESTION TWO (20 MARKS)

a) Construct labeled graphs with 5 vertices (5 marks)

b) Let $U = \{1,2,3,4,5,6,7,8,9,10,11,12\ \}$, $A = \{1,3,5,7,9,11\}$ $B = \{2,3,5,7,11\}$ $C = \{2,3,6,12\}$ and $D = \{2,4,8\}$.

Determine the set

i) $(A \cup B) \cap C^c$

ii) $(C \cap A) \cup D$ (4 marks)

c) In how many ways can 10 people sit at a round table? (3 marks)

d) A committee of twelve is to be chosen from nine men and six women. In how many ways can it be formed if it at least four women are to be on the committee? (4 marks)

e) Show that $\rceil(p \vee q)$ and $\rceil p \wedge \rceil q$ are logically equivalent (4 marks)

QUESTION THREE (20 MARKS)

a) Prove that the square of an even integer is even and the square of an odd integer is odd.
(6 marks)

b) Show that $\sqrt{2}$ is irrational (5 marks)

c) Proof that set of all even natural numbers is countable. (5 marks)

d) Prove the proposition $p(n)$ that the sum of the first n positive integers is $\frac{1}{2}n(n+1)$ that is $p(n) = 1 + 2 + 3 + \ldots + n = \frac{1}{2}n(n+1)$ (4 marks)

QUESTION FOUR (20 MARKS)

A market researcher investigating consumer preference for three brands of soda namely, coke, .fanta and sprite in a certain town gathered the following information; from a sample of 800 consumers 230 took coke 245 took fanta and 325 took sprite. 30 took all the three brands of soda, 70 took coke and sprite, 110 took coke only, 185 took sprite only.

Required;

i) Present the above information in a venn diagram (4 marks)

ii) Determine the number of customers who took fanta only (4 marks)

iii) Determine the number of customers who took coke and fanta only (4 marks)

iv) Determine the number of customers who took fanta and sprite only (4 marks)

v) Determine the number of customers who took none of the brands of soda. (4 marks)

QUESTION FIVE (20 MARKS)

a) Show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent (5 marks)

b) Determine the 37th term of the progression -8, -5, -2, 1,… (5 marks)

c) Determine the value of x for which the following form an A.P. -5, 3, x (5 marks)

d) The first term of a G.P is 5, and the common ratio is -3, which term is equal to -135? (5 marks)