



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

SCO 111/SIT 192: DIFFERENTIAL CALCULUS

DATE: 24/3/2021

TIME: 2.00-4.00 PM

INSTRUCTION: Answer Question *ONE* which is compulsory and any other *TWO* Questions

QUESTION ONE (30 MARKS)

- a) Determine the gradient of the curve $x^2 + 2xy - 2y^2 + x = 2$ at the point $(-4, 1)$ (4 marks)
- b) Determine the inflection point of $f(x) = x^3 - 6x^2 + 9x + 1$ (4 marks)
- c) State the *L'Hôpital's Rule*, hence or otherwise evaluate

$$\lim_{x \rightarrow -3} \frac{x+3}{x^2+5x+6} \quad (3 \text{ marks})$$

- d) Determine $\frac{dy}{dx}$ given that

i. $y = x^x$

ii. $y = (x^2 + 3x)^7$ (6 marks)

- e) Given $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{x}{1-x}$ Determine $(f \cdot g)^{-1}$ (4 marks)

- f) Prove that $\frac{d}{dx}(\sin x) = \cos x$ from the first principles the derivative of (4 marks)

- g) Determine the equation of the tangent line to the curve $y = x^3$ at $(1, 1)$ (2 marks)

- h) Given that $f(0) = 8$, $g(0) = 5$, $f'(0) = 3$, $g'(0) = 1$, Find $F'(0)$ where $F(x) = \frac{f(x)}{g(x)} + 3x^2 + 4$ (3 marks)

QUESTION TWO (20 MARKS)

- a) Determine the values of the gradients of the tangents drawn to the circle $x^2 + y^2 - 3x + 4y = -1$ at $x = 1$ correct to two significant figures (6 marks)
- b) Using logarithmic differentiation, differentiate the following functions;
- i. $xe^x \sin x$ (3 marks)
- ii. $te^t \cos t$ (3 marks)
- c) Determine $\frac{dy}{dx}$ given that $x = \frac{t}{1+t}$, and $y = \frac{t^3}{1+t}$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ (4 marks)
- d) Find the equation of the normal line to the hyperbola $y = \frac{3}{x}$ at the point $x = 3$ (4 marks)

QUESTION THREE (20 MARKS)

- a) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (3 marks)
- b) Obtain $\frac{df(x)}{dx}$ for $f(x) = \frac{\sin x + e^{2x}}{\sin x}$ (4 marks)
- c) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of $5\text{cm}^2/\text{s}$. Find the rate of change of the radius when the area is 30cm^2 (5 marks)
- d) If $y = 3e^{2x} \cos(2x - 3)$, Verify that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$ (8 marks)

QUESTION FOUR (20 MARKS)

- a) An object moves along a coordinate line, its position at each time $t \geq 0$ given by $f(t) = 3t^2 - 7t + 4$. Find the position, velocity and acceleration at time $t = 4$ sec. (3 marks)
- b) Determine $\frac{dy}{dx}$ at $x = 3.1$ given that $y = \frac{2x^3}{\cosh 3x}$ (5 marks)
- c) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through $(1,0)$ and has a stationary point at $(1,0)$. Find the value of α , β , the other turning points and hence sketch the curve (12 marks)

QUESTION FIVE (20 MARKS)

- a) State the mean value theorem of differential calculus and hence determine the value of the constant “c” that satisfy the theorem in $f(x) = x^3 + 2x^2 - x$, $[-1,2]$ (6 marks)
- b) Determine the dimensions that would minimize the total surface area of an open rectangular container if it is to have a volume of $32m^3$ (8 marks)
- c) Determine the domain for each of the following functions
- i) $y = x^3 + 3x - 6$ (1 mark)
- ii) $y = \frac{1}{x^2 + 6x + 9}$ (2 marks)
- d) Given $f(x) = 3x - 2$, determine $f^{-1}(x)$ (3 marks)