MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 202: LINEAR ALGEBRA I

DATE: 20/1/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES

Answer question **ONE** (**compulsory**) and any other **TWO** questions

QUESTION ONE (30 MARKS)

a) Given that $\begin{pmatrix} x+y & 4s+2r \\ x-y & 2s-8r \end{pmatrix} = \begin{pmatrix} 16 & 22 \\ 12 & 14 \end{pmatrix}$ solve for $x, y, s\ and\ r$ (4 marks)

b) Determine $10\boldsymbol{A} + 2\boldsymbol{B}$ Given that $\boldsymbol{A} = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 & 7 \\ -2 & 4 \end{pmatrix}$ (2 marks)

c) Let $\boldsymbol{A} = \begin{pmatrix} 6 & 4 & 9 \\ 3 & -2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 5 & 1 \\ 1 & 7 & 1 \end{pmatrix}$

Compute;

i. $\boldsymbol{A}^2$ (3 marks)

ii. $\boldsymbol{AB}$ (3 marks)

d) Reduce the matrix

e) $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 2 & 0 \\ -1 & -2 & -1 & 1 \\ 1 & -1 & 3 & 3 \end{pmatrix}$ to echelon form. Hence state the Rank of the matrix. (4 marks)

f) Compute the determinants of the following matrices;

i. $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ (2 marks)

ii. $\begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{pmatrix}$ (4 marks)

iii. Given $\boldsymbol{A} = \begin{pmatrix} \lambda & \lambda \\ 6 & \lambda - 4 \end{pmatrix}$ find λ if determinant of $\boldsymbol{A}$ is zero. (4 marks)

g) Solve the equations below using elimination method;

$x_1 + x_2 + x_3 = 5$

$x_1 + 2x_2 + 3x_3 = 10$

$2x_1 + x_2 + x_3 = 6$ (4 marks)

QUESTION TWO (20 MARKS)

a) Given that $\boldsymbol{Q} = \begin{pmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{pmatrix}$ find the minor and co – factors of each element of $\boldsymbol{Q}$

(10 marks)

b) Let $\boldsymbol{R} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix}$. Compute the adjoint of matrix $\boldsymbol{R}$. (10 marks)

QUESTION THREE (20 MARKS)

a) Solve the following system of equations using Gauss – Jordan elimination method.

$x_1 - x_2 + x_3 + 2x_4 = 1$

$2x_1 - x_2 + 3x_4 = 0$

$-x_1 + x_2 + x_3 + x_4 = -1$

$x_2 + x_4 = 1$ (10 marks)

b) Use Crammer's Rule to solve the system of equations below;

$2x + 3y - z = 1$

$3x + 5y + 2z = 8$

$x - 2y - 3z = -1$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Find the displacement vector from the point P with coordinates $(1 + 2\boldsymbol{i}, 1 - 2\boldsymbol{i})$ to the point Q with coordinates $(3 + \boldsymbol{i}, 2\boldsymbol{i})$ (3 marks)

b) Find $\|\boldsymbol{y}\|$ if $\boldsymbol{y} = [1 \quad 2 \quad 3 \quad 4]$ (3 marks)

c) If $\boldsymbol{z} = [1 \quad 0 \quad -3 \quad 2 \quad -1]$ find its normalized vector. (4 marks)

d) Find $\boldsymbol{x}$ if $3\boldsymbol{x} + 2\boldsymbol{y} = \boldsymbol{b}$ where $\boldsymbol{y} = \begin{bmatrix} 3 \\ 1 \\ 6 \\ 0 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 2 \\ -1 \\ 4 \\ 1 \end{bmatrix}$ (4 marks)

e) Find the inverse of **B** i.e. $\boldsymbol{B}^{-1}$ given the matrix; $\boldsymbol{B} = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix}$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Find the inverse of matrix $\boldsymbol{Q}$ given that; $\boldsymbol{Q} = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 2 & 0 \\ 3 & 1 & 1 \end{pmatrix}$ using row-reduction to echelon form of an augmented matrix (8 marks)

b) Consider the following matrices;

$\boldsymbol{S} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $\boldsymbol{T} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ find: $(\boldsymbol{S} + \boldsymbol{T})^2 - (\boldsymbol{S}^2 + 2\boldsymbol{ST} + \boldsymbol{T}^2)$ (8 marks)

c) Show that the matrix equation

$$\begin{bmatrix} 1 & 1 & -2 \\ 2 & 5 & 3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \\ 5 \end{bmatrix}$$

is equivalent to the vector equation $u \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + v \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} + w \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \\ 5 \end{bmatrix}$ (4 marks)