MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

SCO 111/SIT 192: DIFFERENTIAL CALCULUS

DATE: 24/3/2021 **TIME: 2.00-4.00 PM**

INSTRUCTION: *Answer Question* ***ONE*** *which is compulsory and any other* ***TWO*** *Questions*

QUESTION ONE (30 MARKS)

a) Determine the gradient of the curve $x^2+2xy-2y^2+x=2$ at the point (-4, 1) (4 marks)

b) Determine the inflection point of $f(x)=x^3-6x^2+9x+1$ (4 marks)

c) State the $L'H\hat{o}pital's\ Rule$, hence or otherwise evaluate

$$\lim_{x\to -3} \frac{x+3}{x^2+5x+6}$$

(3 marks)

d) Determine $\frac{dy}{dx}$ given that

i. $y=x^x$

ii. $y=(x^2+3x)^7$ (6 marks)

e) Given $f(x)=\frac{x}{x+1}$ and $g(x)=\frac{x}{1-x}$ Determine $(f.g)^{-1}$ (4 marks)

f) Prove that $\frac{d}{dx}(\sin x)=\cos x$ from the first principles the derivative of (4 marks)

g) Determine the equation of the tangent line to the curve $y=x^3$ at (1,1) (2 marks)

h) Given that $f(0)=8,\ g(0)=5,\ f'(0)=3, g'(0)=1$, Find $F'(0)$ where $F(x)=\frac{f(x)}{g(x)}+3x^2+4$

(3 marks)

QUESTION TWO (20 MARKS)

a) Determine the values of the gradients of the tangents drawn to the circle $x^2 + y^2 - 3x + 4y = -1$ at $x = 1$ correct to two significant figures (6 marks)

b) Using logarithmic differentiation, differentiate the following functions;

i. $xe^x \sin x$ (3 marks)

ii. $te^t \cos t$ (3 marks)

c) Determine $\frac{dy}{dx}$ given that $x = \frac{t}{1+t}$, and $y = \frac{t^3}{1+t}$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ (4 marks)

d) Find the equation of the normal line to the hyperbola $y = \frac{3}{x}$ at the point $x = 3$ (4 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (3 marks)

b) Obtain $\frac{df(x)}{dx}$ for $f(x) = \frac{\sin x + e^{2x}}{\sin x}$ (4 marks)

c) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of $5cm^2/s$. Find the rate of change of the radius when the area is $30cm^2$ (5 marks)

d) If $y = 3e^{2x}\cos(2x - 3)$, $Verify\ that$ $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$ (8 marks)

QUESTION FOUR (20 MARKS)

a) An object moves along a coordinate line, its position at each time $t \geq 0$ given by $f(t) = 3t^2 - 7t + 4$. Find the position, velocity and acceleration at time $t = 4$sec. (3 marks)

b) Determine $\frac{dy}{dx}$ at $x = 3.1$ given that $y = \frac{2x^3}{\cosh 3x}$ (5 marks)

c) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through (1,0) and has a stationary point at (1,0). Find the value of α, β , the other turning points and hence sketch the curve (12 marks)

QUESTION FIVE (20 MARKS)

a) State the mean value theorem of differential calculus and hence determine the value of the constant "c" that satisfy the theorem in $f(x) = x^3 + 2x^2 - x$, [-1,2] (6 marks)

b) Determine the dimensions that would minimize the total surface area of an open rectangular container if it is to have a volume of $32m^3$ (8 marks)

c) Determine the domain for each of the following functions

i) $y = x^3 + 3x - 6$ (1 mark)

ii) $y = \dfrac{1}{x^2 + 6x + 9}$ (2 marks)

d) Given $f(x) = 3x - 2$, determine $f^{-1}(x)$ (3 marks)