

MACHAKOS UNIVERSITY
University Examinations 2022/2023
SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
THIRD YEAR FIRST SEMESTER SUPPLEMENTARY EXAMINATION FOR THE
DEGREE OF
BACHELOR OF EDUCATION SCIENCE
BACHELOR OF EDUCATION ARTS
SMA 336: ORDINARY DIFFERENTIAL EQUATIONS II

INSTRUCTION:

Answer Question ONE which is compulsory and any other TWO Questions

QUESTION ONE 30 MARKS

- a) Consider the differential equation $x^2(1-x)y'' + (1-x)y' + y = 0$
- i) Determine the singular points (2 marks)
- ii) Determine the nature of the singular points in (i) above (4 marks)
- b) Solve the system of differential equations below using matrix method (6mks)

$$\frac{dy}{dt} - 2y + 3x = 0$$

$$\frac{dx}{dt} + 2y + 3x = 0$$

- c) Consider the initial – value problem $\frac{dy}{dx} = 2x^2 + y^2, y(1) = 3$.

Determine whether the differential equation has a unique solution (6 marks)

- d) Prove that the Bessel function of order zero is given by

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots \quad (6\text{mks})$$

- e) Show that the total differential equation $y^2 dx - z dy + y dz = 0$ is integrable (6mks)

QUESTION TWO (20 MARKS)

- a) Consider the differential equation

$$\frac{d^4y}{dx^4} - 3\frac{d^3y}{dx^3} + 2\frac{dy}{dx} - 8y = 0$$

Reduce the equation to a system of first order differential equation (5marks)

- b) Consider the system below

$$\frac{dx_1}{dt} = x_1 - x_2 - x_3$$

$$\frac{dx_2}{dt} = x_2 + 3x_3$$

$$\frac{dx_3}{dt} = 3x_2 + x_3$$

Use matrix method to solve the system (15marks)

QUESTION THREE (20 MARKS)

- a) Show that $y = e^{2x}$ is a solution of $(2x + 1)y'' - 4(x + 1)y' + 4y = 0$ (5mks)

- b) Find a linearly independent solution of $(2x + 1)y'' - 4(x + 1)y' + 4y = 0$ by reducing the order

(15mks)

QUESTION FOUR (20 MARKS)

- a) Solve the nonlinear equation $x^2y\frac{d^2y}{dx^2} + \left(x\frac{dy}{dx} - y\right)^2 - 3y^2 = 0$ (10mks)

- b) Prove that the Legendre polynomial of order 3 is given by $P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$ (5mks)

- c) Use Rodrigues formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ to find $P_2(x)$ (5mks)

QUESTION FIVE (20 MARKS)

- a) Show that the equation below has a regular singular point at $x = 0$

$$2x^2y'' - xy' + (1 - x^2)y = 0 \quad (5mks)$$

- b) Solve the above equation in ascending powers of x (15mks)