MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER SUPPLEMENTARY EXAMINATIONS FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF ARTS IN MATHEMATICS AND ECONOMICS

SMA 360: MULTIVARIATE STATISTICAL METHODS 1

DATE: 22/7/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Attempt Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) A random vector $x=[x_1, x_2, x_3]'$ has a variance-covariance matrix given by

$\sum=\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. Find:

i. The variance of $Y=2x_1-x_2$ (3 marks)

ii. The covariance matrix of $z=[z_1, z_2]'$ where $z_1=x_1+2x_2+x_3$ and

$z_2=3x_1+x_2-x_3$ (3 marks)

b) A random vector X has the joint probability density function given by

$f(x_1, x_2, x_3)=\begin{cases} px_1x_2x_3, for, 1\le x_1\le 3, 1\le x_2\le 3, 1\le x_3\le 2 \\ o, elsewhere \end{cases}$. Determine:

i) The value of p (4 marks)

ii) $E(x_1^2x_2x_3)$ (3 marks)

iii) $E(2x_1+3x_2^3+4x_3^2)$ (3 marks)

c) Let $x=[x_1,x_2,x_3]'$ be normally distributed with mean [3,4,5] and $\sum=\begin{bmatrix}3&1&1\\1&4&5\\1&5&1\end{bmatrix}$. Find the conditional density of x_2 given $x_1=2, x_3=6$. (4 marks)

d) List the three properties of Moment Generating Function (M.G.F). (3 marks)

e) Given $M(t_1,t_2,t_3)=[\frac{1}{5}e^{t_1}+\frac{1}{4}e^{t_2}+\frac{3}{10}e^{t_3}+\frac{2}{3}]^{20}$. Find the Moment Generating Function of:

i) x_2 (2 marks)

ii) x_1x_3 (2 marks)

f) Suppose that x_1,x_2,x_3 are random variables having a joint probability density function given by $f(x_1x_2x_3,x_4)=\begin{cases}16x_1x_2x_3x_4, 0\le x_i\le 1\\ 0, elsewhere\end{cases}$. Determine the marginal distribution of: x_2x_4 (3 marks)

QUESTION TWO (20 MARKS)

a) The moment generating function of $x=[x_1,x_2]'$ is given by $m(t_1,t_2)=\left[(1-p_1-p_2)+p_1e^{t_1}+p_2e^{e^{t_2}}\right]^n$. Find the correlation coefficient between x_1 and x_2. (10 marks)

b) Briefly explain what is meant by quadratic form of $x_1,x_2,\cdots x_n$. Hence given that $x=(x_1,x_2,x_3)'$ has a mean vector and variance-covariance matrix given by $\mu=\begin{bmatrix}3\\2\\1\end{bmatrix}$ and $\sum=\begin{bmatrix}5&2&3\\2&3&0\\3&0&2\end{bmatrix}$ respectively. Find the expected of the quadratic form

$Q=(x_2-x_3)^2+(x_1-x_2+x_3)^2$ (10 marks)

QUESTION THREE (20 MARKS)

a) Show that $Var(ax_1 + bx_2) = a^2\delta_{11} + 2ab\delta_{12} + b^2\delta_{22}$ (5 marks)

b) Suppose $x = [x, y]'$ has a two variate normal distribution with density $f(x) = c\exp-\frac{1}{2}Q$.

Where $Q = \frac{1}{7200}(1600x^2 + 900xy + 1125y^2 + 4140x + 450y + 5800)$. Identify μ

(5 marks)

c) Suppose $\underset{\sim}{X} = [x_1 x_2 \cdots x_n]'$ has a continuous distribution defined by

$$f(x) = \begin{cases} \left(\frac{1}{2\pi}\right)^{n/2} \exp - \frac{1}{2}\sum x_i^2 \\ 0, elsewhere \end{cases}$$

. Find the MGF of $\underset{\sim}{X}$. (5 marks)

d) Let x have the variance covariance matrix given by $\sum = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$. Find the

correlation matrix of x (5 marks)

QUESTION FOUR (20 MARKS)

a) Suppose that there are three random variables x_1, x_2, x_3 having the joint PDF

$$f(x_1, x_2, x_3) = \begin{cases} \frac{1}{3}(x_1 + 2x_2 + 3x_3) \\ 0, otherwise \end{cases}$$

. Determine:

i) The marginal PDF of x_1 and x_2. (3 marks)

ii) The conditional density of x_3 given $x_1 = \frac{1}{4}$ and $x_2 = \frac{3}{4}$. (4 marks)

iii) Find the probability $p(x_3 < \frac{1}{2} \mid x_1 = \frac{1}{4}, x_2 = \frac{3}{4})$. (3 marks)

b) If $\underline{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim N\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}\begin{pmatrix} \delta_1^2 & \delta_{12} \\ \delta_{21} & \delta_2^2 \end{pmatrix}\right]$. Write down the PDF of $\underline{X}$. (10 marks)

QUESTION FIVE (20 MARKS)

a) Suppose $\underset{\sim}{X} \sim N(0,1)$. Find the characteristic function of $\underset{\sim}{X}$. (4 marks)

b) Given $Q(t_1,t_2,t_3)=\left[\frac{1}{6}e^{t_1}+\frac{1}{12}e^{t_2}+\frac{1}{3}e^{t_3}+\frac{5}{12}\right]^{12}$. Find

i. $E(x_1)$ (2 marks)

ii. $Cov(x_1,x_3)$ (4 marks)

c) Suppose $x_1,x_2,...,x_n$ are independent Bernoulli random variables with parameter θ. Determine the distribution of $y=\sum_{j=1}^{n}x_j$. (4 marks)

d) Let x_1,x_2,x_3 be independent standard normal random variable. Using the change of variable technique determine the probability distribution of $\underset{\sim}{y}=\left[y_1 y_2 y_3\right]$ where $y_1=x_1$, $y_2=x_1+x_2$ and $y_3=x_1+x_2+x_3$. (6 marks)