MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING.

ECU107: ENGINEERING MATHEMATICS IV

DATE: **TIME:**

INSTRUCTION TO CANDIDATES:

ANSWER **QUESTION ONE** AND **ANY TWO** OTHER QUESTIONS.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Show that the curved surface area of a cone radius r, height h and a slant height l is πrl (3 marks)

b) Determine the inflection point of $f(x) = x^3 - 6x^2 + 9x + 1$ (4 marks)

c) Determine $\frac{dy}{dx}$ given that $y = t^2 - 2t$ and $x = 4 - t^3$ (4 marks)

d) Determine Taylors series expansion generated by the function $f(x) = \ln x$ up to the fifth term about $x = 2$ (4 marks)

e) Find the volume of a hemisphere of radius r units. Hence show that the volume of the sphere of radius r is given by $V = \frac{4}{3}\pi r^3$. (5 marks)

f) Determine the values of the gradients of the tangents drawn to the circle $x^2 + y^2 - 3x + 4y = -1$ at $x = 1$ correct to two significant figures (5 marks)

g) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of $5cm^2 / s$. Find the rate of change of the radius when the area is $30cm^2$ (5 marks)

QUESTION TWO (20 MARKS)

a) An object moves along a coordinate line, its position at each time $t \geq 0$ given by $f(t) = 3t^2 - 7t + 4$. Find the position, velocity and acceleration at time $t = 4$sec.

(3 marks)

b) Determine $\frac{dy}{dx}$ given that

i. $y = x^x$

ii. $y = (x^2 + 3x)^7$ (5 marks)

c) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through (1,0) and has a stationary point at (1,0). Find the value of α, β , the other turning points and hence sketch the curve (12 marks)

QUESTION THREE (20 MARKS)

a) Calculate the volume obtained by rotating the area under the curve $y = 1 + x$ between $x = 1$ and $x = 2$ about the axis of x (4 marks)

b) Using Taylors series, solve

$5xy' + y^2 - 2 = 0, \quad y(4) = 1.$ (16 marks)

QUESTION FOUR (20 MARKS)

a) A projectile is fired straight upwards with a velocity of $400m/s$; its distance above the ground t sec. after being fired is given by $S(t) = -16t^2 + 400t$. $S(t)$ is the distance of the particle from the ground after it has been fired. Calculate the maximum height achieved by the particle (3 marks)

b) Evaluate $\int \sin(3x - 1)dx$ (3 marks)

c) Calculate the maxima and minima values of the function $y = x^3 - 4x^2 - 3x + 2$ and distinguish between them. (5 marks)

d) Evaluate $\int \frac{3x + 7}{(x-1)(x^2+1)} dx$ (9 marks)

QUESTION FIVE (20 MARKS)

a) If $xy + y^2 = 1$ find;

i.) $\frac{dy}{dx}$ (2 marks)

ii.) $\frac{d^2y}{dx^2}$ (2 marks)

b) Evaluate $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (3 marks)

c) Determine the dimensions that would minimize the total surface area of an open rectangular container if it is to have a volume of $32m^3$ (8 marks)

d) A cylinder is to be constructed so that the sum of height and ball radius is 6cm. denoting ball radius by rcm and volume Vcm^3. Show that $V = \pi(6r^2 - r^3)$. Hence show that the value of r which makes V a maxima. (10 marks)