MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SMA 465: PROBABILITY AND MEASURE THEORY

DATE: 7/5/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY)(30 MARKS)

a)

i) State the axioms of a measure, μ.

ii) Show that the function , **m**, defined on the the Borel algebra, B , by:
. **m** : ($\Re$, B) → ($\Re$, B) .such that **m** $\{(a_i,b_i]\} = b_i - a_i$, with a_i, b_i in $\Re$, $a_i \leq b_i$, for i=1,2,…. , is a measure.

(6 marks)

b) Explain, for a given measure space **(Ω, Π, μ)**,

i) Completion of the measure space

ii) **μ** -a.e

iii) the Lebesgue-Stieltjes measure space (6 marks)

c) State:

i) The Lebesgue Monotone Convergence Theorem. (6 marks)

ii) Fatou's Lemma (8 marks)

d) Prove any one of the results stated in Part (c). (4 marks)

e) Let P be a Probability measure. If **A** and **B** are some two events,

i) Define the conditional probability of **A** given **B**.
ii) Show that the conditional Probability of **A** given **B** is a Probability measure.

(8 marks)

QUESTION TWO (20 MARKS)

Let (∩, Π , p) be a probability measure. Assume A_j, j=1,2,….. are events in Π.

a) Show that $P^* = \boldsymbol{\alpha}P + (1-\alpha) P$, $0 < \boldsymbol{\alpha} < 1$, is a probability measure. (5 marks)

b) If A_{j+1} C A_j , j = 1,2,…

then $P(\bigcap A_n) = \lim_{n\to\infty} P(A_n)$ (5 marks)

c) If A_j C A_{j+1} , j = 1,2,….

then $P(\bigcup A_n) = \lim_{n\to\infty} P(A_n)$ (5 marks)

d) $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ (5 marks)

QUESTION THREE (20MARKS)

a) Show that for a non-empty set Ω:

i) The Power set P(Ω) is a σ-field
ii) The set {Φ, A } for A C Ω is not a σ-field

(10 marks)

b) Show that if Π is a σ-field then $\bigcap_{i=1} A_i \in \Pi$ for all $A_i \in \Pi$, i = 1, 2, 3,_ (10 marks)

QUESTION FOUR (20 MARKS)

a) Consider two measures **μ** and **η** defined on a measurable space ,(Ω, Π):

i) Define "**μ** is absolutely continuous with respect to **η**"

ii) Explain "**μ** -null"

iii) When is **μ = η** (9 marks)

b) Show that i) for $E \in \Pi$, the function **m** given by $\mathbf{m}(E) = \int_E d\boldsymbol{\mu}$ is a measure

ii) **m** is absolutely continuous with respect to $\boldsymbol{\mu}$ (8 marks)

c) Let **f** an **g** be measurable functions. Show that **f+g** is measureable (3 marks)

QUESTION FIVE (20 MARKS)

a) State the Chebychev's inequality. (5 marks)

b) Prove the Chebychev's inequality. (5 marks)

c) Let X_n: $n \geq 1$ be a sequence of random variables with $E(X_n) = \boldsymbol{\mu}$,

for all X_n: $n \geq 1$. Show that the sample mean $\underline{\bar{X}}_n$ converges to $\boldsymbol{\mu}$ in probability.

(10 marks)