MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SPECIAL/SUPPLEMENTARY BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE) & BACHELOR OF EDUCATION (SCIENCE), THIRD YEAR

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: 26/7/2019 **TIME: 2:00 – 4:00 PM**

<u>INSTRUCTIONS:</u> Answer Question ONE and any other TWO questions.

<u>QUESTION ONE (30 MARKS)</u>

a) State the second order linear partial differential equation (P.D.E.) in its general form. Hence name the three classical P.D.Es that are the canonical representations of this general 2nd order linear P.D.E. (5 marks)

b) Show that the set of functions $\left\{\sin\frac{n\pi x}{L}\right\}$ is orthogonal with respect to $w(x)=1$ on $[-L,L]$ for $n \neq m$ (5 marks)

c) Write the Fourier series expansion of a function $f(x)$ over $[-L,L]$. (3 marks)

d) Reduce the Laplace equation defined in a rectangular domain $D=\{(x,y)|0\leq x\leq a, 0\leq y\leq b\}$ to a set of ordinary differential equations. (5 marks)

e) Given that $u=u(x,y)$, apply the change of variables $\theta=\theta(x,y)$, $\eta=\eta(x,y)$ to express the second derivatives u_x and u_{yy} in terms of the variables θ,η. (5 marks)

f) Let $f(x),\ g(x)$ be functions whose Fourier transform is $F(w),\ G(w)$ respectively. Let $h(x)$ be the convolution of $f(x),\ g(x)$ such that its Fourier transform $H(w)=F(w)G(w)$. Show that

$$h(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} g(\tau)f(x-\tau)d\tau$$ (3 marks)

g) In solving the boundary value problem $\begin{aligned} -(p(x)y'(x))' + q(x)y(x) = f(x), 0 < x < 1 \\ y(0) - h_0 y'(0) = 0 \\ y(1) - h_1 y'(1) = 0 \end{aligned}$ one determines a function $G(x;s)$ so that the solution is $y(x) = \int_0^1 G(x;s) f(s) ds$.

i) What is the name given to the above boundary value problems? (1 mark)

ii) State the conditions that must be satisfied for $G(x;s)$ to be called a Green's function for the given boundary value problem. (3 marks)

QUESTION TWO (20 MARKS)

a) Sketch the function $f(x) = \begin{cases} -1 & -L < x < 0 \\ 1 & 0 < x < L \end{cases}$, and hence determine the Fourier series expansion of the function. (9 marks)

b) Classify the partial differential equation $xu_{xx} - xu_{yy} - 2u_y = 0$, and transform the P.D.E. to its canonical form. (11 marks)

QUESTION THREE (20 MARKS)

a) Use the appropriate Fourier transform to solve the given boundary-value problem $\begin{aligned} & u_{xx} + u_{yy}, 0 < x < \pi, y > 0 \\ & u(0, y) = f(y), \left.\frac{\partial u}{\partial x}\right|_{x=\pi} = 0, \ y > 0 \\ & \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, 0 < x < \pi \end{aligned}$ (13 marks)

b) The Laplace transform of a function $u = u(x,t)$ is $L\{u(x,t)\} = \int_0^\infty e^{-st} u(x,t) dt$. Determine the Laplace transform of $\frac{\partial^2 u}{\partial t^2}$. (7 marks)

QUESTION FOUR (20 MARKS)

a) Solve the following initial boundary value problem for temperature in a homogeneous isotropic rod with insulated sides and no internal heat generation: $\begin{aligned} & u_t = ku_{xx}, 0 < x < \pi, t > 0 \\ & u_x(0,t) = u_x(\pi,t) = 0, t > 0 \\ & u(x,0) = x, 0 < x < \pi \end{aligned}$ to obtain the particular solution using separation of variables method. The given boundary conditions imply that both ends of the rod are also insulated. (15 marks)

b) Under what assumptions on f, f', f'' does the following hold: $\tilde{F}_S\{f''(x)\} = \int_0^\infty f''(x)\sin\alpha x dx = -\alpha^2 F(\alpha) + \alpha f(0)$ (5 marks)

QUESTION FIVE (20 MARKS)

a) The displacement of a semi-infinite elastic beam is described

$$u_{tt} = u_{xx}\ ,\ x > 0, t > 0$$

by $u(0,t) = t$, $\lim_{x\to\infty} \frac{\partial u}{\partial x} = 0$, $t > 0$. Solve for $u(x,t)$ by the Laplace transform method.

$$u(x,0) = 0,\ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0,\ t > 0$$

(10 marks)

b) Find the characteristics of the equation $u_{xx} + u_{xy} - 2u_{yy} - 3u_x - 6u_y = 9(2x - y)$ and reduce it to the appropriate canonical form. (10 marks)