MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 333: FLUID MECHANICS 1

DATE: 10/8/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (COMPULSORY)(30MARKS)

a) A kilogram of boiling water is mixed slowly in a reversible way with $3kg$ of ice water. Calculate the resultant temperature and the change in enthalpy. (4 marks)

b) Determine the equation of streamline passing through the point $(a, 0)$ if the 2-D flow is described by; $u = \frac{-y}{b^2}$, $v = \frac{x}{a^2}$ (4 marks)

c) Given the velocity fields;

$$V_r = 4\cos\theta\left(1 - \frac{1}{r^2}\right), \qquad V_\theta = -4\sin\theta\left(1 + \frac{1}{r^2}\right), \qquad V_z = 0$$

Show that the velocity field represent a possible incompressible flow (5marks)

d) Prove that $C_p - C_v = \rho$ (5 marks)

e) Calculate the velocity field of a spherically symmetric flow whose stream function is $\psi(r,\theta) = \frac{Ua^3}{2r}\sin^3\theta - \frac{Ur^2}{2}\sin^2\theta$ (5 marks)

f) The x component of the velocity in a certain plane flow depends on y by the relationship $u(y) = Ay$. Determine the y component $v(x, y)$ of the velocity if $v(x,0) = 0$

(5 marks)

g) Determine whether the fluid with vorticity component $u = \dfrac{-2xyz}{(x^2 + y^2)^2}$, $v = \dfrac{x^2 - y^2}{(x^2 + y^2)^2}$,

$w = \dfrac{y}{x^2 + y^2}$ is irrotational. (5 marks)

QUESTION TWO (20MARKS)

a) Determine the analytical function $f(z) = u + iv$ if $u = x^3 - 3y^2 + 3x^2 - 3y^2 + 2x + 1$

(10 marks)

b) If $w = \phi + i\psi$ represent the complex potential for an electric field and

$\psi = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$.

Determine the function ϕ. (10 marks)

QUESTION THREE (20MARKS)

a) The ideal gas obeying Boyle's law $P = k\rho$ is rushing from a boiler through a conical pipe with radii a and b; $(b > a)$ at the two ends. If the gas enters a pipe from the narrow end with the velocity V and escapes through the other end with velocity U, show that

$V = \dfrac{Ub^2}{a^2}\exp\left[\dfrac{V^2 - U^2}{2k}\right]$ (12 marks)

b) If ϕ and φ are functions satisfying Laplace equations. Show that $s + it$ is analytic function where;

$s = \dfrac{\partial\phi}{\partial y} - \dfrac{\partial\varphi}{\partial x}$, $t = \dfrac{\partial\phi}{\partial x} - \dfrac{\partial\varphi}{\partial y}$ (8 marks)

QUESTION FOUR (20MARKS)

A two dimensional flow towards a normal boundary is found to be characterized by a normal component of the velocity that varies directly with distance from the boundary. Determine;

i.) The stream function (11 marks)

ii.) The stream line (9 marks)

QUESTION FIVE (20MARKS)

a) Show that for a diabetic process $P\rho^{-r} = cons\tan t$; where $r = \frac{Cp}{Cv}$ (8 marks)

b) Expressing the enthalpy S and internal energy E in terms of pressure P and volume V, show that;

i.) $\left(\frac{\partial S}{\partial P}\right)_V = \frac{C_V}{\alpha kT}$ (6 marks)

ii.) $\left(\frac{\partial S}{\partial V}\right)_P = \frac{C_P}{\alpha VT}$ (6 marks)

$\alpha - coefficient\ of\ volume\ \exp ansion$

$k - Bulk\ \mathrm{mod}\, ulus$