



# MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 262: DIFFERENTIAL EQUATIONS

DATE: 25/3/2021

TIME: 2.00-4.00 PM

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## INSTRUCTIONS

*Answer question ONE and any other TWO questions*

### QUESTION ONE 30 MARKS (COMPULSORY)

- a) Define
- i. Differential equations
  - ii. Order of a differential equation
  - iii. Ordinary differential equation (3 marks)
- b) Show that the function  $y = \cos x$  is a solution of the ordinary differential equation defined by  $\frac{d^2y}{dx^2} + y = 0$  (3 marks)
- c) Form a differential equation from the primitive defined by  $y = Ax^2 + Bx$  (5 marks)
- d) solve the differential equation defined by  $\frac{dy}{dx} = \frac{x^2-2}{y}$  (5 marks)
- e) Find the orthogonal trajectories of the curves whose equation is  $x^2 - y^2 = c$  (5 marks)
- f) Solve the differential equation defined by  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$  (5 marks)
- g) Define the particular solution of  $\frac{d^2y}{dx^2} + 4y = \cos 3x$  (4 marks)

**QUESTION TWO (20 MARKS)**

- a) Find the orthogonal trajectories of a curve whose equation is  $y = \frac{x}{c+x}$  (5 marks)
- b) Form a differential equation from the primitive defined by  $y = A \cos 2x + B \sin 2x$  (5 marks)
- c) A substance cools from 100°C to 60°C in 10 seconds. Find the temperature of the substance after 40 seconds assuming that the room temperature is 20°C. Given that  $\frac{dT}{dt} \propto (T - T_0)$ . With  $T$  being the temperature at time  $t$  and  $T_0$  being the room temperature (10 marks)

**QUESTION THREE (20 MARKS)**

- a) Find the particular solution of the ordinary differential equation given by  

$$\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = x - 2$$
 (6 marks)
- b) Find the solution of the differential equation defined by  $\sin x \cos y \, dx + \cos x \sin y \, dy = 0$  (4 marks)
- c) Using the method of undetermined coefficients solve the differential equation defined by  

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x^2$$
 (10 marks)

**QUESTION FOUR (20 MARKS)**

- a) solve the linear differential equation defined by  

$$\frac{dy}{dx} - \frac{y}{x} = x^2$$
 (6 marks)
- b) Show that the solution of the differential equation defined by  
 $\cos y \sin x \, dx + \sin y \cos x \, dy = 0$  is  $\cos x \cos y = A$  (7 marks)
- c) Use Laplace transform to solve the differential equation defined by  

$$\frac{dy}{dx} - 5y = 0 ; y(0) = 2$$
 (7 marks)

**QUESTION FIVE (20 MARKS)**

- a) Determine whether the differential equation below is exact and find the solution of the differential equation  

$$2xy \, dx + (1 + x^2) \, dy = 0$$
 (7 marks)
- b) Show that the solution of an homogeneous linear differential equation with roots that are real and equal is given by  

$$y = (Ax + B)e^{\alpha x}$$
 (4 marks)
- c) Solve the following differential equation  

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - \frac{2}{x}y = \ln x$$
 (9 marks)