



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING
EEE 402: COMPLEX ANALYSIS FOR ENGINEERING

DATE: 23/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS

Answer question ONE and any other TWO questions

Programmable calculators are prohibited

QUESTION ONE (30 MARKS)

- a) Distinguish between
- Single valued function and many valued function (give example in each case) (4 marks)
 - Continuity in a domain and uniform continuity (2 marks)
 - Analytic function and entire function (2 marks)
- b) Define limit of a complex function $f(z)$ and hence evaluate $\lim_{z \rightarrow i} f(z) = z^2 + 2z$ (4 marks)
- c) Find the image of the point $(4, 3)$ on a z -plane under the transformation $w = 2z^2 + 3$ (4 marks)
- d) Evaluate the points of discontinuity of
- $f(z) = \frac{3z^2 + 1}{z^2 + 1}$ (4 marks)

ii. $f(z) = \frac{z^2 + 1}{z^4 - 16}$ (4 marks)

e) Show that $f(z) = -2xy + i(x^2 - y^2)$ is analytic (6 marks)

QUESTION TWO (20 MARKS)

- a) State the condition for integrability of $f(z)$ (2 marks)
- b) State and prove the Cauchy's integral formula and give the general expression for the n th derivative of $f(z)$ (7 marks)
- c) Evaluate using Cauchy's integral formula $\oint_c \frac{z^2 - 4z + 4}{z + i} dz$, where c is the circle $|z| = 2$ (4 marks)
- d) Integrate z^2 along the straight line OM (direct) and along an indirect path consisting of two straight line segments OL and OM, where O is the origin, M is the point $z = 3 + i$ and L (3,0). Show that integral of z^2 along the two paths are equal. Hint: Sketch the region (7 marks)

QUESTION THREE (20 MARKS)

- a) Let $w = f(z) = 4z + 4 - 3i$, find the value of w which correspond to $z = -2 - i$ (5 marks)
- b) Evaluate the following integral $\int_c \frac{e^{iz}}{z^2(z-2)(z+5i)} dz$ $|z| = 3$ using residue theorem. (10 marks)
- c) A point $3 + bi$ on a z -plane is mapped onto the point $(11, c)$ on the w -plane by the mapping function $f(z) = 2z^2 + 1$, find the values of a and b (5 marks)

QUESTION FOUR (20 MARKS)

- a) State three conditions for a complex function $f(z)$ to be continuous at a point $z = z_0$ (3 marks)
- b) Examine the continuity of $f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$ (8 marks)
- c) Show that $\cos^{-1}(z) = -i \ln(z + \sqrt{z^2 - 1})$. Hence find all solutions to the equation $\cos z = 2i$ (7 marks)
- d) Use the definition of $\sin z$ and $\cos z$ to prove the following relation
 $1 + \tan^2 z = \sec^2 z$ (2 marks)

QUESTION FIVE (20 MARKS)

- a) Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies Laplace equations (6 marks)
- b) Find all the values of z such that $e^z = 1 + i\sqrt{3}$ (5 marks)
- c) Find the poles of $f(z) = \frac{z^3 + 5z + 1}{z - 2}$ (5 marks)
- d) Find $f(z) = u + iv$, given that $f(z)$ is analytic and $u = x^3 - 3x^2y$ (4 marks)