



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRDYEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 333-FLUID MECHANICS I

DATE: 26/9/2019

TIME: 2:00 – 4:00 PM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE AND ANY TWO OTHER QUESTIONS

QUESTION ONE: COMPUSARY (30 MARKS)

a.) The pressure inside a soap bubble of radius $0.01m$ balances a $6.0m$ column of oil of specific gravity $0.80 \times 10^3 kg / m^3$. Calculate the surface tension of the soap solution. (5 marks)

b.) A fluid is at rest over a horizontal surface. Show that the pressure varies in vertically upward direction is $\frac{\partial p}{\partial z} = \rho g$,

Where ρ is the density of fluid. Hence the pressure at depth h from the top is given by

$$p = p_o + \rho gh,$$

where p_o is the atmospheric pressure at the top surface of the fluid. (5 marks)

c.) State the Bernoulli's Theorem and its significance. Hence or otherwise prove Bernoulli's equation (10 marks)

d.) A simple substance is such that the internal energy U , the pressure p and the volume v of the unit mass are related by

$U = 3pv$. Show that;

$$C_v = 3v \left(\frac{\partial p}{\partial T} \right)_v, \quad C_p = 4p \left(\frac{\partial v}{\partial T} \right)_p.$$

Assuming that $\gamma \left(= \frac{C_p}{C_v} \right)$ is constant, show that

$$p^{3\gamma} v^4 = F(T),$$

where $F(T)$ is a function of temperature. (10 marks)

QUESTION TWO (20 MARKS)

a.) Find the mathematical equation for the force per unit volume acting on the fluid element. (5 marks)

b.) Show that the work done, required to break up a liquid drop of radius R into n equal droplets each of equal size, is given by

$$W = 4\pi R^2 T (n^{1/3} - 1), \text{ Where } T \text{ is the surface tension.} \quad (5 \text{ marks})$$

c.) Derive the equation of continuity for a moving fluid (10 marks)

QUESTION THREE (20 MARKS)

a.) Two circular glass plates each of radius $r = 10\text{cm}$ are pressed together with a liquid film of thickness $d = 2 \times 10^{-6}\text{cm}$. Assuming that the liquid completely wets the glass plates, calculate the force of attraction between the plates. (surface tension = 70 dynes/cm) (4 marks)

b.) Find the velocity of the fluid flowing through Venturi tube. Also determine the mass of liquid flowing through the constricted part. (6 marks)

c.) If the line source of strength m is situated at $z = z_1$, show that the complex velocity potential is given by $w = -m \log(z - z_1)$ (10 marks)

QUESTION FOUR (20 MARKS)

a.) Discuss the flow for which $w = z^2$ (6 marks)

b.) Test whether the motion specified by $\vec{v} = \frac{k^2(xj - yi)}{x^2 + y^2}$ ($k = \text{constant}$) is a possible motion for an incompressible fluid. If so, determine the equation of the streamlines. Also test whether the motion is of the potential kind and if so, determine the velocity potential. (14 marks)

QUESTION FIVE (20 MARKS)

Prove the Euler's equation

$$\frac{\partial \mathbf{v}}{\partial t} + \boldsymbol{\Omega} \times \mathbf{v} = -\nabla \left(\frac{p}{\rho} + \frac{1}{2} v^2 + \psi \right)$$

For fluid element in non-stationary motion. Also explore when the fluid is irrotational, incompressible with steady motion. (20 marks)