MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

First YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

SST 102: DISCRETE MATHEMATICS

DATE: 18/4/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer Question One and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain the meanings of the following terms

(i) a *binary relation* on a set A, (3 marks)

(ii) a *recurrence relation* (3 marks)

b) Given that $S = \{n \in \mathbb{Z}: 1000 < n \leq 3000\}$. Determine then number of elements of S that are divisible by 5 but by not by 3. Justify your answers clearly. (5 marks)

c) Consider the sets $A = \{3,45,201\}$ and $B = \{2,17,45,48,300\}$ and let R be a relation defined from A to B such that for any $a \in A$ and $b \in B$ then aRb if and only if a is greater than B or if a divides b. Determine the elements of R. (5 marks)

d) Let $g: \mathbb{R} \to \mathbb{R}$ be the function defined by $g(x) = x^2 + 1$. Prove that g is neither injective nor surjective. (5 marks)

e) Majority of Kenyan private vehicle number plates have the format $K ** \#\#\# *$ where each character $*$ is alphabetic letter and each character # is a numeric digit (e.g. KAB123C). Determine the number of distinct number plates having this format that can be produced. (5 marks)

f) Determine the number of non-negative integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 121$. (4 marks)

QUESTION TWO (20 MARKS)

a) Explain the meaning of the terms

(i) transitive binary operation on a set A, (2 marks)

(ii) relatively prime numbers $a, b \in \mathbb{Z}$. (2 marks)

b) Define a relation $\sim$ on the set $\mathbb{Z}$ as follows: $a \sim b$ if and only if $a - b$ is divisible by 3.

(i) Prove that $\sim$ is an equivalence relation. (4 marks)

(ii) Determine the equivalence class of 2. (3 marks)

c) Use the Euclidean algorithm

(i) to find $d = \gcd(5613,772)$, (4 marks)

(ii) express $\gcd(5613,772)$ in the form $d = 5613m + 772n$. (5 marks)

QUESTION THREE (20 MARKS)

a) Let a, b and c be integers. Prove that

(i) If $a \mid b$ then $a \mid bx$ for any integer x (3 marks)

(ii) If $a \mid b$ and $b \mid c$, then $a \mid c$. (3 marks)

b) Let a, b, m be integers with $m > 0$ and suppose that $a \equiv b(mod\ m)$.

(i) Prove that $a + x \equiv b + x(mod\ m)$ for any integer x. (4 marks)

(ii) Solve the equation $374x \equiv 51(mod\ 289)$ (5 marks)

c) Each user on a computer system has a password which is seven to nine characters long. Each character in a generated password can be a lowercase alphabetic letter or a numeric digit. Each password must contain at least one digit. Determine the possible number of unique passwords that can be generated. (5 marks)

QUESTION FOUR (20 MARKS)

a) Let $\mathbb{Z}$ denote the set of integers. Consider the functions $f: \mathbb{Z} \to \mathbb{Z}$ and $g: \mathbb{Z} \to \mathbb{Z}$ given respectively by $f(n) = 2n + 3$ and $g(n) = 3n^2 + 2$. What is

(i) the composition $f \circ g$. (2 marks)

(ii) the composition $g \circ f$, (4 marks)

b) Let a, p, q be integers and suppose that $a \mid p$ and $a \mid q$. Prove that $a \mid (up + vq)$ for any $u, v \in \mathbb{Z}$. (4 marks)

c) Suppose that there are 3 distinct courses in computational mathematics, 5 distinct courses in statistics and 8 distinct courses in fluid mechanics. Determine the number of distinct ways can a student choose 2 courses in computational mathematics, 3 courses in statistics and 4 courses in fluid mechanics. (5 marks)

d) Consider the sequence $\{a_n\}$ defined by $a_n = -a_{n-1} + 20a_{n-2}$. Solve this recurrence relation subject to the initial condition $a_0 = 1, a_1 = 14$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Let $\sim$ be a binary relation defined on $\mathbb{R}^2$ by $(a, b) \sim (x, y)$ if and only if $(a, b) = \lambda(x, y)$ for some real number $\lambda \neq 0$.

(i) Prove that $\sim$ is an equivalence relation on $\mathbb{R}^2$. (5 marks)

(ii) Describe the equivalence class of $(1,1)$ under the equivalence relation $\sim$ defined above. (2 marks)

b) Prove that $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$. (5 marks)

c) Assume that a country currently with a population of 40 million people has a population growth of 3%.

(i) Determine how big its population will be after 15 have elapsed. (4 marks)

(ii) If the country receives 200,000 new immigrants every year, determine its population in 15 years' time assuming that all the immigrants arrive at the end of the year and reproduce at the same rate as the native population. (4 marks)