MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY),

BACHELOR OF EDUCATION (SPECIAL NEEDS EDUCATION)

BACHELOR OF EDUCATION (SCIENCE),

SPH 300: WAVE THEORY

DATE: 24/8/2022 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

Answer question **ONE** which is compulsory and any other **TWO questions**

Take acceleration due to gravity, g =10 N/kg

SECTION A (COMPULSORY)

QUESTION ONE (30 MARKS)

a) i. Name two factors that a simple harmonic oscillator must exhibit (2 marks)

ii. Show that $y = A\cos k\,x + B\sin k\,x$ is a general solution to simple harmonic motion (3 marks)

b) Show that the solution in a (ii) can also be written in terms of

i. phase angle α (3 marks)

ii. complex exponential function (3 marks)

c) Derive the equation for the periodic time and frequency of a compound pendulum in simple harmonic motion (3 marks)

d) Explain two applications of Doppler effect in real life (4 marks)

e) Show that energy in a reactive electrical system is sinusoidal (4 marks)

f) i. A 20 g mass hanging from a spring causes an extension of 1 mm. If the mass is displaced and allowed to vibrate freely, calculate the period and frequency of the vibration. (4 marks)

ii. If two similar springs to that in (i) were used in series, how will the period and frequency be affected (2 marks)

QUESTION TWO (20 MARKS)

a) Show that a transverse wave in a vibrating string obeys the wave equation (8 marks)

b) i. Determine the Fourier series for the function $f(x) = x^2$ for $-\pi < x < \pi$ (8 marks)

ii. Use the series to show that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$ (4 marks)

QUESTION THREE (20 MARKS)

a) Distinguish

i. Damped and forced vibrations (2 marks)

ii. Critical and over damping (2 marks)

b) Show that a charged capacitor discharging through an inductor and resistor represents damped harmonic motion (5 marks)

c) A mechanical system of mass m and stiffness constant k experiences forced vibrations of amplitude of the form $A\ cos\omega t$ where ω is the frequency of the driving force. Given that $\omega_0 = \sqrt{\frac{m}{k}}$ and $\beta = \frac{b}{2m}$ for damping constant b, show that :-

i. The phase angle, δ may be expressed as $tan\,\delta = \frac{A}{\omega_0{}^2 - \omega^2}$ (5 marks)

ii. The amplitude of the forced vibrations may be written as $D = \frac{A}{\sqrt{(\omega_0{}^2 - \omega^2)^2 + 4\omega^2\beta^2}}$ (4 marks)

iii. The particular solution as $t \to 0$, for the displacement, $x = D\,cos(\omega t - \delta)$ (2 marks)

QUESTION FOUR (20 MARKS)

a) Distinguish interference and diffraction (4 marks)

b) Show that nth maxima will be produced at X_n given by $X_n{}^2 = \frac{(2n-1)(a+p)p\lambda}{a}$ where a is the distance from the source to the barrier and p is the the distance from barrier to the screen (6 marks)

c) If X is the distance between the first two minima, derive the expression for the wavelength of the light used (4 marks)

d) A narrow slit is illuminated by light of wavelength 589 nm is placed at a distance of 20 cm from a straight edge. If a screen is placed at a distance of 1.7 m from the slit, calculate the distance between the first and the third maxima. (6 marks)

QUESTION FIVE (20 MARKS)

a) A mass on a spring oscillates with an amplitude of 5 cm at a frequency of 1Hz. If at $t = 0$, $x = 0$,

i. Find the equation describing the position of the mass as a function of time in the form $x = A\,cos(\omega t + \alpha)$ (4 marks)

ii. Find the position, velocity and acceleration at t = $\frac{8}{3}$seconds. (3 marks)

b) i. Derive the equation of motion of a simple pendulum of length l when the horizontal displacement of its mass from equilibrium is x for $x << l$. Show that the period $T = 2\pi\sqrt{\frac{l}{g}}$. Find T when l = 2 m (6 marks)

ii. If the 0.1 kg pendulum bob in (i) has an amplitude of 0.02 m, determine the energy of the bob. (7 marks)