MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 807: MODELLING WITH PARTIAL DIFFERENTIAL EQUATION

DATE: 25/1/2022 **TIME: 10.00-1.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) State the four types of traffic flow models (4 marks)

b) State five characteristics of turbulent flows (5 marks)

c) Prove that for an incompressible steady inviscid flow the Bernoulli equation is given by $P + \Omega + \frac{1}{2}v^2 = A$, where A is a constant. (5 marks)

d) Considering Euler's equation of motion. Determine a three dimensional flow equations in components or tensor form (5 marks)

e) A fluid is flowing steadily between two fixed parallel plates under pressure gradient $q = -\frac{\partial p}{\partial x}$ and both plates are stationary. Show that the velocity distribution is $u = \frac{q}{2\mu}\left(d^2 - y^2\right)$ where $2d$ is the distance between the plates and y is measured from the mid-point. (5 marks)

f) Consider a solid sphere of radius a centered at O, moving with a uniform velocity Ui in an incompressible fluid of infinite extent which is at rest in infinity. Derived its velocity potential equation components at the point $p(a,\ \theta,\ \varphi)$ on the surface of the sphere. (6 marks)

QUESTION TWO (20 MARKS)

a) In order to determine the magnitudes of the turbulent shear stress a number of semi-empirical theories have been developed. Discuss any three such theories. (9 marks)

b) Consider the following modelled governing equations;

i. $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

ii. $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\rho g-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2};$$

iii. $$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\rho g-\frac{\partial p}{\partial y}+\mu\frac{\partial^2 v}{\partial y^2}$$

iv. $$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2}$$

Given the dimensionless parameters;

$$X=\frac{x}{L};\quad Y=\frac{y}{L};\quad U=\frac{u}{U_0};\quad V=\frac{v}{U_0};\quad \theta=\frac{T-T_\infty}{T_w-T_\infty}$$

$$R_e=\frac{U_0L}{v},\quad P_r=\frac{v}{\alpha}$$

Non-dimensionalize the equations (i), to (iv) (11 marks)

QUESTION THREE (20 MARKS)

Given a project with the title "steady incompressible non-newtonian fluid flow past an infinite parallel plates ". If the flow is two dimensional and the parallel plates are at $y=0$ *and* $y=d$ with velocities $u=0,$ *and* $u=0$ and maintained at constant temperatures T_1 *and* T_2 respectively. The velocity is a long $x-axis$; all flow variables are functions of y. Develop this project's;

a) Specific velocity profile equations (9 marks)

b) Specific temperature profile equation (7 marks)

c) Expression for the skin frictions at each plate. (4 marks)

QUESTION FOUR (20 MARKS)

a) State five characteristics of laminar flows (5 marks)

b) Prove the Kelvin's circulation theorem (8 marks)

c) Brine of specific gravity 1.15 is flowing from the bottom of a large open tank through a $80mm$ pipe. The drain pipe ends at a point $10m$ below the surface of the brine in the tank and passing through the center of the drain line to the point of discharge. Assuming the friction is negligible. Calculate the velocity of the flow along the stream line at the point of discharge from the pipe. (7 marks)

QUESTION FIVE (20 MARKS)

Consider the insect population dynamics with competition model below;

$$x_{n+1} = f(x_n;\ R_0,\ b) = \frac{R_0 x_n}{(1+x_n)^b}$$

Where;

$R_0 - basic \quad reproductive \quad ratio$

$x_n - insect \quad population$

$b - compensation \quad parameter$

a) Determine the steady state equation for the model for $R < 1 \quad and \quad R > 1$ (12 marks)

b) Show that for $R < 1 \quad and \quad 0 < b \leq 1$ the non-trivial steady state is monotonically stable. (8 marks)