



MACHAKOS UNIVERSITY

University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 302: FINANCIAL MATHEMATICS II

DATE: 18/3/2022

TIME: 11:00 – 1:00 PM

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) 30 MARKS.

- a) GIVEN :
- i. $\delta = 8\%$, calculate i^4 . (2 marks)
 - ii. $i = 7\%$, calculate δ (2 marks)
- b) The current share price of stock is £ 200. Dividends are paid continuously and the current dividend is £ 5 pa. Calculate the forward price of a five year contract on the asset if the risk free force of interest is 5% p.a. (3 marks)
- c) A 7 month loan is repayable by a single period of £ 5000, if the loan is issued at a rate of commercial discount of 7% p.a. How much was the initial amount lent to the borrower. (3 marks)
- d) List five characteristics that are particular to property investments. (5 marks)
- e) A stochastic interest rate model assumes that the interest rates in different years are independent and identically distributed normal random variable with mean 8% and standard deviation 2%. Calculate the mean and standard deviation of the accumulated value at time 2 of an initial investment of £ 10,000. (5 marks)
- f) A ten year bond with half yearly coupons of 6% p.a has just been issued with a redemption yield of 9% p.a effective. It is redeemable at par. What price would an investor paying 15% tax on income pay for the bond? Tax payments are due four months after each coupon is received. (5 marks)

g) Consider a two factor model. if :

- The mean specific return is 1.0%
- The expected values of the two factors are 3.0% and 2.2% and ,
- The sensitivities of investment returns to each of the factors are 0.8 and - 0.3 respectively.

What is the expected return predicted by the model?

(5 marks)

QUESTION TWO (20 MARKS)

An investor has the choice of the following assets that earn rates of the return as follows in each of the four possible states of the world.

State	probability	Asset 1	Asset 2	Asset 3
1	0.2	5%	5%	6%
2	0.3	5%	12%	5%
3	0.1	5%	3%	4%
4	0.4	5%	1%	7%
Market capitalization		10,000	17,546	82,454

Determine the market price of risk assuming CAPM holds and define the terms.

QUESTION THREE (20 MARKS)

a) In a particular bond market, the two-year par yield at time $t = 0$ is 4.15% and the issue price at time $t = 0$ of a two-year fixed interest stock, paying coupons of 8% annually in arrears and redeemed at 98, is £105.40 per £100 nominal. Calculate:

- The one-year spot rate. (8 marks)
- The two-year spot rate. (4 marks)

b) A lump sum of \$14,000 will be invested at time 0 for 4 years at a constant annual rate of interest i . $(1 + i)$ has a log-normal distribution with mean 1.05 and variance 0.007.

What is the probability that the investment will accumulate to more than \$20,000 in 4 years' time? (8 marks)

QUESTION FOUR (20 MARKS)

- a) Find the annual effective interest rate equivalent to a nominal rate of 12% p.a convertible four monthly. (5 marks)
- b) Evaluate the value at time $t = 0$ of £ 400 due at time $t = 7$ and £ 1000 due at time = 10 , if $\delta(t) = 7\%$ for all t. (5 marks)
- c) An investor, who is liable to income tax at 20% but is not liable to capital gains tax, wishes to earn a net effective rate of return of 5% per annum. A bond bearing coupons payable half-yearly in arrear at a rate 6.25% per annum is available. The bond will be redeemed at par on a coupon date between 10 and 15 years after the date of issue, inclusive. The date of redemption is at the option of the borrower. Calculate the maximum price that the investor is willing to pay for the bond. (10 marks)

QUESTION FIVE (20 MARKS)

- a) A fixed interest security pays coupons of 8% p.a half yearly in arrear and is redeemable at 110%. Two months before the next coupon is due, an investor negotiates a forward contract in which he agrees to buy £ 60,000 nominal of the security in six months' time. The current price of the stock is £ 80.40 per £100 nominal and the risk- free force of interest is 5% p.a .calculate the forward price. (5 marks)
- b) Project R delegates all the development work to outside companies. The estimated cash flows for project R are: (last column indicates expenditure)

Beginning of year 1	150,000 dollars	Contractor fees
Beginning of year 2	250,000 dollars	Contractor fees
Beginning of year 3	250,000 dollars	Contractor fees
End of year 3	1,000,000 dollars	Sales

Table 1

Project S carries out all the development work in-house by purchasing the necessary equipment and using the company's own staff. The cash flows for the project S are: (last column indicates expenditure)

Beginning of year 1	325,000 dollars	New equipment
Throughout year 1	75,000 dollars	Staff cost
Throughout year 2	90,000 dollars	Staff cost
Throughout year 3	120,000 dollars	Staff cost
End of year 3	1,000,000 dollars	Sales

Table 2

The staff cost can be estimated to be paid uniformly throughout the year.

- a) Evaluate the NPV for the project R and S using a risk discount rate of 10% pa?
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(13 marks)
- b) Using the NPV as a criterion, which project is preferable?
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(2 marks)