SAC 402: LOSS DISTRIBUTIONS AND CREDIBILITY THEORY

QUESTION ONE (30 MARKS)

a. Individual claim amounts on a portfolio of motor insurance policies follow a gamma distribution with parameters α and ϰ. It is known that ϰ = 0.8 for all drivers, but the value of α varies across the population. Given that α ~ Gamma (200,0.5), calculate the mean and variance of a randomly chosen claim amount. (6 marks)

b. A random variable X is believed to follow an Exp (λ) distribution. In order to test the null hypothesis μ = 20 against the alternative hypothesis μ = 30, where $\mu = \frac{1}{\lambda}$, a single value is observed from the distribution. If this value is less than 28, H_0 is accepted, otherwise H_0 is rejected. Calculate the probabilities of:

   (i) a Type I error (3 marks)

   (ii) a Type II error. (3 marks)

c. In a one-year mortality investigation, 45 of the 250 ninety-year-olds present at the start of the investigation died before the end of the year. Assuming that the number of deaths has a binomial distributions with parameters n=250 and q, calculate a symmetrical 90% confidence interval for the unknown mortality rate q. (6 marks)

d. Let S have a Poisson frequency distribution with parameter λ = 5. The individual claim amount has the following distributions:

x	$f_X(x)$
100	0.8
500	0.16
1000	0.04

Calculate the probability that aggregate claims will be exactly 600. (6 marks)

e. The estimator $\delta,^2$ is used to estimate the variance of a $N(\mu,\delta^2)$ distribution based on a random sample of n observations:

$$\delta,^2 = \frac{1}{n}\sum_{i=1}^{n}(Yi - \bar{y})^2$$

   i. Determine the mean square error of $\delta,^2$

   ii. Determine whether $\delta,^2$ is consistent. (6 marks)

QUESTION TWO (20 MARKS)

a. There are two groups of good and bad drivers in the city. The good drivers make up to 70% of the population of the drivers. The annual claim amount of each good driver is uniformly distributed on (0, 3000). The annual claim for each bad driver is exponentially distributed with mean ϴ = 5,000. As many as 100 drivers are chosen at random. Find the mean and variance of the aggregate claim. (10 marks)

b. Aggregate losses have compound Poisson claim distribution with λ = 3 and an individual claim amount distribution p (1) = 0.4, p (2) = 0.3, p (3) = 0.2 and p (4) = 0.1. Determine the probability that aggregate losses do not exceed 3. (10 marks)

QUESTION THREE (20 MARKS)

a. A random sample of 100 claim amounts $x_1, x_2, \ldots, x_{100}$ is observed from a Weibull distribution with parameter $\gamma = 2$, where c is unknown. For these data:
$\sum Xi$ = 487,926 $\sum Xi^2$ =976,444,000 sample median = 4,500

i. Show that the maximum likelihood estimate for c based on a sample on a sample of size n is given by:

$$\hat{C} = \frac{n}{\sum Xi^2}$$

And hence estimate the value of c. (6 marks)

ii. Estimate the value of c using the method of moments. (2 marks)

iii. Calculate the method of percentiles estimate of c. (2 marks)

b. Losses arising from a portfolio follow a Pareto distribution with parameters $\alpha = 3$ and $\lambda = 2{,}000$. Calculate the probability that a randomly chosen loss amount exceeds the mean loss amount. (5 marks)

c. Claims arising from a certain type of insurance policy are believed to follow an exponential distribution. The lower quartile claim is 200. Calculate the mean claim size. (5 marks)

QUESTION FOUR (20 MARKS)

f. A random sample of n observations is taken from a normal distribution with mean μ and variance δ^2. The sample variance is an observation of a random variable S^2. Using the relationship between the gamma and chi-squared distributions, derive expressions for $E(S^2)$ and $var(S^2)$. (10 marks)

g. The estimator $\delta_,^2$ is used to estimate the variance of a $N(\mu,\delta^2)$ distribution based on a random sample of n observations:

$$\delta_,^2 = \frac{1}{n}\sum_{i=1}^{n}(Yi - \bar{y})^2$$

iii. Determine the mean square error of $\delta_,^2$

iv. Determine whether $\delta_,^2$ is consistent. (10 marks)

QUESTION FIVE (20 MARKS)

An actuary wishes to analyse the amounts paid by a group of insurers on their respective portfolios of commercial property insurance policies using the models of Empirical Bayes Credibility Theory.

The actuary obtains the following information about the amounts of claim payments made and the number of policies sold for each of three different insurers. The data obtained are as follows.

	Year 1	Year 2	Year 3	Year 4
Insurer A	£14.2m	£15.8m	£22.7m	£19.0m
	163	189	252	199
Insurer B	£58.6m	£63.1m	£81.0m	£64.2m
	4,435	4,761	5,576	4,581
Insurer C	£123m	£132m	£161m	£133m
	16,184	17,443	20,102	18,000

i. Analyse the data using EBCT Model 1, and calculate the expected total claim payment to be made by Insurer B in the coming year. (8marks)

(ii) Analyse the data using EBCT Model 2, and again calculate the expected payout amount for Insurer B in the coming year, assuming that the expected number of policies sold for the coming year for Insurer B is 4,800. You may use the summary statistics given below, which have been calculated using the formulae and notation given in the Tables, again working in millions of pounds. Subscripts 1, 2 and 3 refer to Insurers A, B and C respectively (9 marks)

$\sum P_{1,j} (y_{1j} - \bar{Y}_1)^2 = 0.014667$ $\quad$ $\sum P_{1,j} (y_{1j} - \bar{Y})^2 = 5.106461$

$\sum P_{1,j} (y_{2j} - \bar{Y}_2)^2 = 0.006103$ $\quad$ $\sum P_{1,j} (y_{2j} - \bar{Y})^2 = 0.336408$

$\sum P_{1,j} (y_{3j} - \bar{Y}_3)^2 = 0.003979$ $\quad$ $\sum P_{1,j} (y_{3j} - \bar{Y})^2 = 0.292641$

ii. Comment on your results (3 marks)