MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (BIOLOGY)

SMA 101: MATHEMATICS FOR SCIENCE II

DATE: 14/6/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

Answer Question ***ONE*** *which is compulsory and any other* ***TWO*** *Questions*

QUESTION ONE (30 MARKS)

a) At what values of x is the function $f(x)=\dfrac{x^2+x-6}{x^2-4}$ continuous? (3 marks)

b) Determine the area of the region in the first quadrant that is bounded by $y=\sqrt{x}$, $x-axis$ and the line $y=x-2$ (4 marks)

c) State the $L'H\hat{o}pital's\ Rule$ and hence evaluate $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1-(x/2)}{x^2}$ (4 marks)

d) Determine $\frac{dy}{dx}$ given that

i. $y=x^x$

ii. $y=(x^2+3x)^7$ (6 marks)

e) i Given that the domain of the function $y=\sqrt{5-x}$ is $\{-2,-1,3\}$ find its range and Co-domain

ii Given $f(x)=\dfrac{x}{x+1}$ and $g(x)=\dfrac{x}{1-x}$ Determine $(f.g)^{-1}$ (5 marks)

f) Find from first principle the derivative of $y = x\ sin3x$ (3 marks)

g) An object is launched straight up into the air at an initial velocity of 64m/s. six metres off the ground. If its height (H) in metres at t seconds is given by the formula $H = 16t^2 - 64t + 6$. determine

i. The maximum height reached

ii. The time it hits the ground and the velocity at the time (5 marks)

QUESTION TWO (20 MARKS)

a) Determine the values of the gradients of the tangents drawn to the circle $x^2 + y^2 - 3x + 4y = -1$ at $x = 1$ correct to two significant figures (6 marks)

b) Determine the area under the graph of a nonnegative function $y = x$ the $x - axis$ and the lines $x = 1$ and $x = 4$ (6 marks)

c) Determine and identify the stationary values of the function $f(x, y) = x^3 - 6x^2 - 8y^2$

(8 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\lim_{x\to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (2 marks)

b) The region bounded by the curve $y = \sqrt{x}$, $x - axis$ and the line $x = 4$ is revolved about the $x - axis$ to generate a solid. Determine the volume of the solid so formed. (5 marks)

c) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of $5cm^2 / s$. Find the rate of change of the radius when the area is $30cm^2$ (5 marks)

d) If $y = 3e^{2x} \cos(2x - 3)$, $Verify\ that \quad \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$ (8 marks)

QUESTION FOUR (20 MARKS)

a) A 2.6m ladder is placed against a wall such that its foot is one metre from the wall. If the top of the ladder is sliding down the wall at $0.2m / s$, determine the rate at which the ladder is moving away from the wall (4 marks)

b) Find $\frac{dy}{dx}$ at $x = 3.1$ given that $y = \frac{2x^3}{\cosh 3x}$ (6 marks)

c) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through (1, 0) and has a stationary point at (1, 0). determine the value of α , β and the other turning points (10 marks)

QUESTION FIVE (20 MARKS)

a) State the mean value theorem and hence determine the value of the constant "c" that satisfy the theorem in $f(x) = x^3 + 2x^2 - x$, $[-1, 2]$ (6 marks)

b) Determine the dimensions that would minimize the total surface area of an open rectangular container if it is to have a volume of $32m^3$ (7 marks)

c) Evaluate

i. $\int xe^x dx$

ii. $\int_0^{\frac{\pi}{2}} 5\sin 2x dx$ (7 marks)