

MACHAKOS UNIVERSITY
SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
SECOND YEAR, FIRST SEMESTER EXAMINATIONS
For Diploma in Education

SMA 0205: Vector Analysis

Date

Time

Instructions

Attempt question **one** and any other **two** questions.

Show all your working.

QUESTION ONE 30MKS

a) Define a i) Scalar product (2m)

ii) Vector product (2m)

b) Given that $\mathbf{P} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{Q} = 2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$ evaluate

i) $\mathbf{P} \cdot \mathbf{Q}$ (3m)

ii) $\mathbf{P} \times \mathbf{Q}$ (4m)

c) If $\mathbf{A} = (u+3)\mathbf{i} - (2u+u^2)\mathbf{j} + 2u^3\mathbf{k}$ determine

i) $\frac{d\mathbf{A}}{du}$ ii) $\frac{d^2\mathbf{A}}{du^2}$ (4m)

d) If $F = 2uv\mathbf{i} + (v^2-5u)\mathbf{j} - (2u+v^2)\mathbf{k}$ determine

i) $\frac{\partial F}{\partial u}$ ii) $\frac{\partial^2 F}{\partial u^2}$ iii) $\frac{\partial^2 F}{\partial u \partial v}$ (6m)

e) i) Given $\phi = 2x^2y^3z$. Find $\text{grad } \phi$ (3m)

ii) Show that $\text{curl } (-y\mathbf{i} + x\mathbf{j})$ is a constant vector. (6m)

QUESTION TWO 20MKS

a) Given $\mathbf{A} = x^2y^3\mathbf{i} - 2xyz^2\mathbf{j} + x^2z\mathbf{k}$

$\mathbf{B} = xy^2z\mathbf{i} + 3yz^2\mathbf{j} - xyz^2\mathbf{k}$ and that

$\phi = xy^2z^3 - 3xy^2 + xyz^2$

Determine at the point (1, -2, 1)

i) $\nabla \phi$

ii) $\nabla \cdot \mathbf{A}$

iii) $\nabla \times \mathbf{B}$ (10m)

b) If $\mathbf{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{OB} = \mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ determine

i) the value of $\mathbf{OA} \cdot \mathbf{OB}$ (2mks)

ii) the product $\mathbf{OA} \times \mathbf{OB}$ in terms of unit vectors (2mks)

iii) the cosine of the angle between $\mathbf{OA}$ and $\mathbf{OB}$ (6mks)

QUESTION THREE 20MKS

a) If $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{B} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, and $\mathbf{C} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$,

Determine

i) $\mathbf{A} \cdot \mathbf{B}$ (2mks)

ii) angle between $\mathbf{A}$ and $\mathbf{B}$ (3mks)

iii) $\mathbf{B} \times \mathbf{C}$ (3mks)

b) If $\mathbf{A} = x^2y\mathbf{i} + (xy + yz)\mathbf{j} + xz^2\mathbf{k}$

$\mathbf{B} = yz\mathbf{i} - 3xz\mathbf{j} + 2xy\mathbf{k}$ and

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$ determine at the point (1,2,1)

i) $\text{grad } \phi$ (3mks)

ii) $\text{div grad } \phi$ (3mks)

iii) $\text{grad div } \mathbf{A}$ (3mks)

iv) $\text{div curl } \mathbf{B}$ (3mks)

QUESTION FOUR 20MKS

a) Given that $u = e^x[\sin(y + z) - y\cos(y + z)]$

Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + u = 2e^x \sin(y + z)$ (10mks)

b) Find the directional derivatives of the function $\phi = x^2z + 2xy^2 + yz^2$ at the point

(1,2, -1) in the direction of the vector $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ (10mks)

QUESTION FIVE 20MKS

A surface consists of five sections formed by the planes $x = 0$, $x = 1$, $y = 0$, $y = 3$,

$Z = 2$ in the first octant. If the vector field $F = y\mathbf{i} + z^2\mathbf{j} + xy\mathbf{k}$ exists over the surface and around its boundary verify Stokes theorem.