MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND AND THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
BACHELOR OF SCIENCE IN MATHEMATICS

SST 201, SMA 362: OPERATIONS RESEARCH I

DATE: 21ST JANUARY, 2025 **TIME:2.00 PM – 4.00 PM**

INSTRUCTIONS:

1. *Answer **Question** **1** and any other **two** questions.*
2. *You must have a Scientific Calculator and Graph Paper.*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

(a) Explain briefly *four* conditions which must be satisfied for a problem to be solved using linear programming technique. (4 *marks*)

(b) Explain each of the following types of variables as used in linear programming, in the context of both an initial feasible solution and an optimum solution:

(i) Slack variable;
(ii) Surplus variable. (6 *marks*)

(c) Given the linear programming model below:

$$\text{Minimise :} \quad z = 10x_1 + 8x_2 + 15x_3$$

$$\text{Subject to :} \quad 3x_1 + 2x_2 + 5x_3 \geq 30$$

$$7x_1 + 4x_2 + 6x_3 \geq 45$$

$$2x_1 + 5x_2 + 3x_3 \geq 60$$

$$\text{With :} \quad x_1, \; x_2, \; x_3 \geq 0$$

Derive its symmetrical dual program. (2 *marks*)

(d) A grain grocery sells two types of grains: beans and groundnuts. Beans is sold at Ksh 100 per tin, while groundnuts is sold at Ksh 150 per tin. The owner of the grocery wants to determine the daily production plan which maximises contribution to sales revenue. The production constraints are as shown in the table below:

	Machine hours	Labour hours	Storage expenses	Packaging material
Beans	4	1	2	1
Groundnuts	3	2	1	1
Maximum available	480	160	180	100

A customer has placed a standing order for the supply of a minimum of 20 tins of groundnuts per day for further processing for export.

(i) Formulate a linear programming model for the problem. (3 *marks*)

(ii) Using the graphical method, determine the optimum production plan which maximises the daily sales revenue for the grocery. (6 *marks*)

(iii) Interpret the optimum solution obtained in (ii) above. (3 *marks*)

(e) A shipping company is required to meet the demands at various destinations by transporting 80, 100, 140, 60, 120 containers respectively from various supply sources with 120, 180, 70, 130 containers. The transport cost in thousand Kenya shillings per unit of container over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 4 & 8 & 7 & 2 & 6 \\ 8 & 6 & 5 & 4 & 2 \\ 6 & 2 & 1 & 3 & 5 \\ 5 & 1 & 2 & 4 & 3 \end{bmatrix}$$

The company wants to meet the demand at the destinations by transporting the containers at the cheapest cost possible. Derive the *initial basic solution* and its corresponding total cost of transport using each of the following methods:

(i) Northwest corner (NWC) rule;

(ii) Vogel's approximation method (VAM). (6 *marks*)

QUESTION TWO (20 MARKS)

A restaurant sells three types of hot drinks – tea, cocoa and coffee. The sale prices per cup of the drinks are: tea Ksh 40, cocoa Ksh 25 and coffee Ksh 50. The daily production plan for the sales revenue has been modelled as an LP program as shown below.

Maximise :	$z = 40x_1 + 25x_2 + 50x_3$	
Subject to :	$2x_1 + 3x_2 + x_3 \leq 1200$	fuel
	$x_1 + x_3 \leq 450$	labour hours
	$2x_1 + 4x_3 \leq 600$	sugar
	$x_2 \leq 150$	sales agreement
With :	$x_1, x_2, x_3 \geq 0$	non-negativity conditions

where x_1 – the number of cups of tea produced,
x_2 – the number of cups of cocoa produced, and
x_3 – the number of cups of coffee produced.

The restaurant wishes to determine the daily production plan which maximises contribution to its sales revenue.

(i) Using the Simplex method, determine the optimum daily production plan for the restaurant.

(16 *marks*)

(17 Interpret the optimum solution obtained in (i) above. (4 *marks*)

QUESTION THREE (20 MARKS)

Below is a linear programming model for a given problem. Use it to answer the questions that follow.

Minimise :	$z = 120x_1 + 140x_2 + 80x_3$
Subject to :	$x_1 + 2x_2 + x_3 \geq 30$
	$2x_1 + x_2 + x_3 \geq 45$
With :	$x_1, x_2, x_3 \geq 0$

(i) Determine the *symmetrical dual* of the linear programming model above. (2 *marks*)

(ii) Using the Simplex method, determine the optimum solution of the dual program for the linear programming model above. (10 *marks*)

(iii) Using the relationship between a primal program and its symmetrical dual program, extract the optimum solution of the primal program from the optimum solution of its dual program. (4 *marks*)

(iv) Interpret the optimum solution extracted in (iii) for the primal program of the linear programming problem. (4 *marks*)

QUESTION FOUR (20 MARKS)

A transport company distributes cement which is required to meet the demands 50, 70, 40, 20 trailers at various destinations from supplies 60, 20, 100 trailers from various sources. The transport cost per unit amount of fuel consumed in thousand litres over the various routes are as given in the cost matrix below:

$$C = \begin{bmatrix} 3 & 2 & 5 & 7 \\ 4 & 1 & 6 & 1 \\ 8 & 5 & 15 & 9 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the cement at the minimum fuel consumption possible. Using the northwest corner (NWC) rule, do the following for the transportation problem:

(i) Determine the optimal solution that minimises the fuel consumption; (17 *marks*)

(ii) Interpret the optimum solution obtained in (i) above. (3 *marks*)

QUESTION FIVE (20 MARKS)

(a) (i) Outline *four* characteristics of an assignment problem as used in operations research. (4 *marks*)

(ii) Outline any *four* sources of constraints in an optimisation problem involving linear programming. (2 *marks*)

(b) A computer networking company has its network technicians based at different locations in the county to maintain the networks at the premises of their various customers. Four maintenance requests have been received, and four technicians are available on standby. The distance in kilometres each of the technicians is away from the various customer premises is as shown in the table below.

		Customer			
		C1	C2	C3	C4
Technician	T1	26	24	14	16
	T2	15	23	19	24
	T3	22	24	14	20
	T4	21	18	22	17

The company intends to assign the technicians to the customers in such a way that the total distance to be travelled is minimised. Determine the optimal assignment of the

technicians to the customer premises that will minimise the total distance travelled. (14 *marks*)