MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE/STATISTICS & PROGRAMMING/MATHEMATICS)
SMA 261: PROBABILITY AND STATISTICS II

DATE: TIME:

<u>Question One. Compulsory (30 marks)</u>

(a) If the joint probability function for x_1 and x_2 is given by $f(x, y) = \frac{1}{30}(x + y)$

$x = 0, 1, 2, 3.$
$y = 0, 1, 2$

(i) Show that $f(x, y)$ is a probability distribution (3 marks)

(ii) Hence or otherwise compute $p(x \leq 2)$ (3marks)

(b) Given that $f(x, y) = \begin{cases} \frac{3}{2}y^2, & 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine the marginal probability function for x and y (6marks)

(c) Consider the bivariate table below

		y		
		-1	0	1
X	-1	0.04	0.01	0.20
	1	w	0.03	0.60

(i) Determine the value of w (2marks)

(ii) Determine the marginal density of x (3marks)

(d).The joint pdf of x and y is given by $f(x,y) = \boldsymbol{b}xy, for\ 0 < x, y < 1$.
Determine the value of $\boldsymbol{b}$ (4marks)

(e).Show that E(X+Y)=E(X)+E(Y) if X and Y are independent continuous random variables (5marks)

(f).If x and y are independent random variables with a joint p.d.f defined by:

$$f(x,y) = \begin{cases} 4xy & 0 < \mathrm{x}, \mathrm{y} < 1, \\ 0 & \text{elsewhere} \end{cases}$$

Determine the E(x, y) (4marks)

Question Two

(a) The joint probability density function for x and *y* is given by $f(x,y) = \frac{1}{54}(x+y)$
x = 1, 2, 3.
y = 1, 2,3,4

(i) Determine the conditional distribution of y given that X=x (5marks)
(ii) Hence calculate $p(y = 4/x = 3)$ (3 marks)

(b).Suppose that x and y are discrete random variables with a p.d.f as shown in the bivariate table below:

		y		
		0	1	2
x	0	0.1	0.1	0.2
	1	0	0.15	0.05
	3	0.1	0.2	0.1

Determine the covariance of x and y ie cov(x,y). (12marks)

Question Three

(a) Given x and y have the joint probability density function (pdf) given by

$$f(x,y) = \begin{cases} K(x^2 + y^2), & 0 \le x \le 1; 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

Determine (i) the value of the constant K. (3 marks)
(ii) the conditional p.d.f of x given for any value of y (5marks)

(iii) $pr(x < \frac{1}{2}/y = \frac{1}{2})$ (2marks)

(b).Suppose X is a continuous and Y is discrete random variables with a joint p.d.f given by:

$f(x,y) = \frac{yx^{y-1}}{3}$,$0 < x < 1, y \in 1,2,3$.Determine

(i) $f_y(y)$ (5marks)

(ii) $E(X)$ (5marks)

Question Four

(a) The joint pdf of x and y is given by $f(x,y) = \begin{cases} 2e^{-(x+2y)}, x > 0, y > 0 \\ 0 \quad \text{elsewhere} \end{cases}$

Determine whether or not x and y are independent. (6 marks)

(b) The joint pdf of x and y is given by that $f(x,y) = \begin{cases} 6x^2y, & 0 < \mathrm{x} < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine: (i) μ_x *and* μ_y (4 marks)

(ii) d_x^2 *and* d_y^2 (4 marks)

(iii) The correlation coefficient between x and y (6 marks)

Question Five

(a) Given that xand y are independent random variables with a joint pdf defined by that

$f(x,y) = \begin{cases} 1, & 0 < x, y < 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine the joint probability density function of $v = x + y$ *and* $v = x - y$. (10 marks)

(b) Given that E(y/x)=$\propto + \beta x$, and $E(X/Y) = \varphi + \delta y$ where E denotes expectation, x and y denotes random variables and $\propto$, β,φ, δ are constants. Show that:

$\frac{\propto}{\varphi} = \frac{1-\beta}{1-\delta}$ (10 marks)