



# MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 202: MODELS IN ACTUARIAL MATHEMATICS

DATE: 30/8/2022

TIME: 2.00-4.00 PM

## INSTRUCTIONS

Answer question one (**compulsory**) and any other **two** questions

### SECTION A: ATTEMPT ALL QUESTIONS (30 MARKS)

#### QUESTION ONE (30 MARKS)

- a) Suppose that there is a “uniform distribution of deaths” from age  $x$  to age  $x + 1$ ,

i.e  $l_{x+t}$  is linear in  $t$  for  $0 \leq t \leq 1$ . Show that, for all  $0 \leq s \leq t \leq 1$

$${}_{t-s}q_{x+s} = \frac{t-s}{1-s} q_x \quad (3 \text{ marks})$$

- b) Suppose that  ${}_t p_x \mu_{x+t}$  is decreasing for  $0 \leq t \leq 1$ . Show that  $q_x < \mu_x$  (3 marks)

- c) Evaluate (a)  $\overline{A}_{60}$  and (b)  $A_{60}$

on the basis of (i) A1967-70 ultimate, 4% p.a interest

(ii) E.L.T. No. 12-Males, 4% p.a interest. (4 marks)

- d) i) Show that  $A_x = vq_x + vp_x A_{x+1}$  (4 marks)

ii) Given that  $p_{60} = 0.985$ ,  $p_{61} = 0.98$ ,  $i = 0.05$  and  $A_{62} = 0.6$ , evaluate  $A_{61}$  and  $A_{60}$

(2 marks)

- e) Using commutation functions or otherwise calculate the values of the following
- (i)  $A_{1\overline{30}|20}$  on A1967-70 Ultimate ,4% (2 marks)
- (ii)  $A_{1\overline{30}|20}^-$  on A1967-70 Ultimate ,4% (2 marks)
- f) (i) Write down an expression for  $a_x$  in terms of  $v$ ,  $p_x$ , and  $a_{x+1}$  (1 mark)
- (ii) On a certain select mortality table the select period is one year. Express  $a_{[x]}$  in terms of  $v$ ,  $p_{[x]}$  and  $a_{x+1}$ . Given that  $q_{[x]} = 0.6 q_x$  and that at  $4\frac{1}{4}\%$  p.a interest  $a_{45} = 15.719$  and  $a_{46} = 15.509$ , find the value of a  $a_{45}$  (at the same rate of interest). (5 marks)
- g) A 5- year temporary assurance, issued to a woman aged 55, has a sum assured of \$ 50,000 in the first year, reducing by \$10,000 each year. The sum assured is payable at the end of the year of death. Level premiums, limited to at most 3 years' payments, are payable annually in advance. Calculate the annual premium.
- Basis:
- A1967-1970 select mortality
- 4% p.a interest
- Expenses are 10% of all office premiums (4 marks)

## **SECTION B: ANSWER ANY TWO QUESTIONS (40 MARKS)**

### **QUESTION TWO (20 MARKS)**

- a) A special whole life assurance policy is issued on a life aged 50 exact. Under this policy the sum assured, payable at the end of the year of death, is 1 unit for the first 10 years decreasing to 0.75 units thereafter.
- (i) Calculate the expected present value of the benefit. (4 marks)
- (ii) Determine the variance of the present value of the benefit. (5 marks)
- Basis: Mortality AM92 Ultimate Interest 4% per annum
- b) Calculate
- i.  $\ddot{a}_{40:\overline{4}|}$  (2 marks)
- ii. Derive the value of  $A_{1\overline{40}|4}$  Using your result from part (i). (5 marks)

Basis: From the following life table extract

| x  | $\ell_x$ |
|----|----------|
| 40 | 100,000  |
| 41 | 99,200   |
| 42 | 98,100   |
| 43 | 96,700   |
| 44 | 94,700   |

Interest rate is 5%

- c) By considering a term assurance policy as a series of one-year deferred term assurance policies, show that:

$$A_{x:\overline{n}|}^- = \frac{i}{\sigma} A_{x:\overline{n}|} \quad (5 \text{ marks})$$

### QUESTION THREE (20 MARKS)

- a) A life aged 50 who is subject to the mortality of the A1967-70 Select table, effects a pure endowment policy with a term of 20 years for a sum assured of £10,000.
- Write down the present value of the benefits under this contract, regarded as a random variable. (2 marks)
  - Assuming an effective rate of interest of 5% per annum, calculate the mean and the variance of the present value of the benefits available under this contract. (4 marks)
- b) A life aged exactly 60 wishes to arrange for a payment to be made to a charity in 10 years' time. If he is still alive at that date the payment will be £1000. If he dies before the payment date, the amount given will be £500. Assuming an effective interest rate of 6% per annum and mortality according to ELT No.12-Males, calculate the standard deviation of the present value of the liability. (6 marks)
- c) Let X be a random variable representing the present value of the benefits of a whole of life assurance, and Y be a random variable representing the present value of the benefits of a temporary assurance with a term of n years. Both assurances have a sum assured of 1 payable at the end of the year of death and were issued to the same life aged x.
- Describe the benefits provided by the contract which has a present value represented by the random variable  $X - Y$ . (1 mark)

- ii. Show that:

$$\text{Cov}(X, Y) = {}^2A_{\overline{x:n}} - A_x A_{\overline{x:n}}$$

and hence or otherwise that:

$$\text{Cov}(X - Y) = {}^2A_x - ({}_nA_x)^2 - {}^2A_{\overline{x:n}}$$

Where the functions A are determined using an interest rate of  $i$ , and functions  ${}^2A$  are determined using an interest rate of  $i^2 + 2i$

#### QUESTION FOUR (20 MARKS)

- a) Write down a single term expression to represent the present value of a deferred annuity-due of 1 pa payable to a life now aged  $x$ , with a deferment period of  $n$  years. Hence derive a formula for the variance of the present value of this contract (13 marks)
- b) i) Find the present value of a deferred annuity of £1000 p.a. to a man aged 40. Payments commence on his 60th birthday, if he is then alive, and continue thereafter annually for life.  
Basis: A1967-70 ultimate mortality, 4 % p.a. interest. (2 marks)
- ii) As above, but select mortality (at entry.) (2 marks)
- iii. As above, but A1967-70 ultimate mortality to exact age 60 and a(55) males ultimate mortality above exact age 60. (3 marks)

#### QUESTION FIVE (20 MARKS)

- a) A life insurance company issues 20-year temporary assurance policies to lives aged 45. The sum assured, which is payable immediately on death, is £400,000 for the first 10 years, and £100,000 thereafter. Level annual premiums are payable in advance for 20 years, or until earlier death. The premium basis is: Mortality: AM92 Ultimate Interest: 4% per annum Expenses: nil.
- i. Show that the premium payable is approximately £870.25 per annum. (4 marks)
- ii. Find the net premium reserve ten years after the commencement of the policy, immediately before the payment of the eleventh premium, assuming the reserving basis is the same as the premium basis. (4 marks)
- iii. Give an explanation of your numerical answer to part (ii). Describe the disadvantages to the insurance company of issuing this policy. (3 marks)

- iv. How could the terms of the policy be altered, so as to remove the disadvantages described in part (iii)? (2 marks)
- b) Let  $g(T)$  be the present value of the profit to the life office, at the issue date, in respect of an  $n$ -year without profits endowment assurance to  $(x)$  with sum assured (payable immediately on death if this occurs within  $n$  years) and premium  $P$  per annum, payable continuously for the term of the policy. Expenses are ignored in all calculations.
- i. Write down an expression for  $g(T)$ . (1 mark)
- ii. Derive expressions for (1) the mean, and (2) the variance of  $g(T)$ . (4 marks)
- iii. For what value of  $P$  is the mean of  $g(T)$  equal to zero? (2 marks)